Magnetic Reconnection in the Solar-Terrestrial program

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and a lot more...

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Frozen-in theorem

Let's consider $K$ along the magnetic field $B$ : $K \times B = 0$

For an ideal Ohm's law $E = -V_i \times B$, one can show $d_t(K \times B) = 0$

→ The magnetic field is frozen in the plasma : they are co-moving together & keep coupled

A particle cannot jump from a magnetic field line to another : no reconnection
deHoffmann-Teller frame

Non-relativistic Lorentz transform: \[ E' = E + V_{R'/R} \times B \]

One can define \( V_{HT} = (E \times B)/B^2 \): In this frame, the (perpendicular) electric field is null, so particle keep gyrate around a same field line through time.

In the \( HT \) frame, a magnetic field line is non-moving.

For ideal MHD: \( V_{HT} = V_\perp \) so plasma & magnetic field are frozen one into the other.

To allow \( d_t(K \times B) \neq 0 \), one needs \( \nabla \times E_\parallel \neq 0 \)
Reconnexion in ideal MHD?

In ideal MHD, magnetic connections are conserved, except eventually where $B = 0$

- Either the electric field is null, so lines of forces do not cross separatrices
- Or the plasma velocity must be discontinuous, for $E \neq 0$, which is forbidden by its inertia
Reconnection rate: definition

Amount of magnetic flux across a closed surface: \( \int B \, ds \)

The reconnection rate is the time derivative of this quantity

\[ \frac{\partial}{\partial t} \int B \, ds \]

Using Faraday eq. & Stokes theorem, the reconnection rate is \( \mathbf{E} = -\partial_t A \)
Ohm’s law (electron momentum equation)

\[
E = -V_i \times B + \frac{1}{qn} J \times B + \frac{m}{q} \frac{d}{dt} V_i - \frac{m}{nq^2} \frac{d}{dt} J - \frac{1}{nq} \nabla \cdot P_e + \eta J - \eta^* \Delta J
\]

(1) : ideal term iscales like \( V_i/V_A \)
(2) : Hall effect scales like \( kl_p \)
(3) : electron inertial effect scales like \( \omega/\Omega_e \)
(4) : electron inertial effect iscales like \( kl_p \omega/\Omega_e \)
(5) : electron compressibility scales like \( k \rho_L e v_{the}/V_A \)
(6) : resistivity (dissipative term)
(7) : hyper-viscositA (dissipative… depends on scale)
What if 3D?

Asks Sophie Masson...

Very large scales phenomenon, needing a MHD description

→ Micro-Physics can hardly be considered because of scales discrepancies
→ Plasms weakly collisional close to the photosphere
→ Bounded problems where boundary conditions are important

• Numerical works are oftenly using resistive MHD
Fast reconnection killed by small Lundqvist nbr ?

In Sweet-Parker model, reconnection rate scales as $S^{-1/2}$: → reconnection should be quenched in collisionless media...

*Loureiro et al, 2012*
2D reconnection: out-of-plane electric field

1. Ideal MHD: \( E = -V_i \times B \)
2. \( p^+ \) Diff. region: \( E = (J \times B)/en \)
3. \( e^- \) Diff. region: \( E = -\nabla \cdot P_e/en \)
Fast Reconnection: Hall effect [GEM Challenge, 2001]

→ Hall effect governs the reconnection rate
Numerical simulation of plasmas

Self-consistant electromagnetic fields:

\[ \mathbf{B} \text{ from } \partial_t \mathbf{B} = -\nabla \times \mathbf{E} \text{ (always)} \]

- in MHD: \( \mathbf{E} = -\mathbf{V}_i \times \mathbf{B} \)
- in two-fluid & hybrid: complete Ohm’s law \( (\mu_0 \mathbf{J} = \nabla \times \mathbf{B}) \)
- in full-PIC: \( c^{-2} \partial_t \mathbf{E} = \nabla \times \mathbf{B} - \mu_0 \mathbf{J} \) (+ Poisson correction)

→ So need density and current density from plasma equations (or ion flow in MHD)
3 possible approaches

• MHD and 2-fluid: set of fluid equations + closure (need an hypothesis) and eq. on $E$

• Hybrid: $p^+$ as macroparticles $\rightarrow$ kinetic effects, $e^-$ as a massless fluid (closure)

• full-PIC: both $p^+$ & $e^-$ are macroparticles (strong constraints on mass ratio & $c/V_A$)

$\rightarrow$ Vlasov codes are unaffordable in 3D...
Set of equations for hybrid models

\[ \begin{align*}
\frac{d_t x_i}{} & = v_i \\
\frac{d_t v_i}{} & = E + v_i \times B - \eta J \\
\partial_t B & = -\nabla \times E \\
J & = \nabla \times B \\
E & = -V_p \times B + N^{-1}(J \times B - \nabla \cdot P_e) + \eta J \\
P_e & = NT_e
\end{align*} \]

→ Electric field has electrostatic component
→ no need of Poisson correction (and Laplacian to invert)

- How to define \( n \) and \( V_p \) from collections of \( x_i \) & \( v_i \)
How to manage macro-particles?

- A macro-particle is representative of a set of particles...
- In a statistically acceptable way (nrb of part per cell)
- A macro-particle has finite size, more or less “diffusive”
- The size of the macro-particle depends on the mesh size
- Macro-particles flow one through the other

→ Fluid moments depend on assignment function
How to manage macro-particles?

Shape factor:

$S(x)$ are \textit{b-splines} of order 1, 2, 3... or more?

\[
N(x) = \sum_{i=0}^{N-1} S(x - x_i)
\]

\[
V_p = \sum_{i=0}^{N-1} S(x - x_i)v_i/N
\]

→ Only defined on grid points

→ and convolution product for 2- and 3-dimensions
→ bounce motion of the ions is associated to the Hall electric field (electrostatic) and not to the magnetic field
Energy budget: Aunai et al., 2011

→ Thermal energy is larger than bulk energy (outflow)
Reconnexion mediated by High Power lasers: t=0
When folding targets [Smets et al., 2014]

Initial out-of-plane magnetic field: Quadripolar structure
→ Reconnection rate depends on sallient/reverse angle
→ 6 shots scheduled on LMJ/PETAL: spring 2019
Reconnected flux

\[
\rightarrow B_Z \text{ develops prior the reconnection onset (t=16)}
\]

\[
\rightarrow \text{Same reconnection rate at each loci (slope of } A_Z)\]

\[
\rightarrow \text{Time lag between the 2 onsets of reconnection}\]
Local X-line orientation: [Aunai et al., 2016]

→ Reconnection rate depends on the 2D simulation plane
→ It is maximum in the plane defined by bisector of upstream fields
Coming soon: PHARE

Parallel Hybride code using Adaptive-Mesh-refinement

Developed across LPP & LERMA:
Space plasmas, Lab. astrophysics, ISM...

Small team (3 persons) hopefully growing

→ aims at being state-of-the-art for Petascale... Exascale

1D version beginning of 2019, 2D end of 2019,
and then 3D...
Which kind of AMR?

- It could have been “cell-based” (see P. Kestner & P4est)
  - Nice because refined only in the needed cells
  - But we have particles to manage...
  - and self-forces appear for $\partial_t S(x) \neq 0$

- So it is “patch-based”
  - refine in a given patch (of given geometry)
  - a (refinement) level is a collection of patches
  - eases the parallelization
What about self-forces?

Multi-Level-Multi-Domain techniques:

*Innocenti et al., 2013*

- A coarse level exists for the WHOLE simulation domain
- Patches exist at finer levels (with finer particles)
  → the entry flux is dictated by next coarser level
  so a refinement operator exists for BOTH fields & particles
  → the outgoing flux is... outgoing!
- No $\partial_t S(x)$, only fine living patches
  → Make sure levels are “physically coherents”
What else?

- The electric field coming from Ohm’s law, no simple leap-frog
  → Predictor-Predictor-Corrector scheme *kuntz et al., 2013*

- Yee grid

- Non-relativistic *Boris* pusher (80% of CPU)
  → Abstract base classes as interface for various solver, grid, electron closure, *b-spline* order, dimension...
Written in C++17

FOR(mula)TRAN(slation) tuned for numerical operation...
Now, most of the code is software engineering.

C++ combines low-level optimized code &
high-level abstract code

smart pointers (memory leaks), STL for containers...
of course, data encapsulation, polymorphism, inheritance
Design pattern

“general reusable solution to a commonly occurring problem within a context in software design”

Needed to code factoring, avoid bugs, ease lecture...

Strategy, Factory...

Interpolator for 1, 2, 3D and 1, 2, 3 & 4th order?
→ Bridge
Projet management & Documentation

- Redmine:
  provides issues, doc, wiki, Gantt, calendar, forum, files...

- Doxygen:
  generate the documentation (for developers) automatically from comments in the code

UML diagrams are also included
Unit, integration & validation Tests (GoogleTest)

- Unit tests : aims at testing a "unitary component", basically a method, or a class

- Integration tests : be sure that a whole chain of components is well integrated

- Validation tests :
  - functional... a given function is fulfilled
  - "solution"... the solution of a given problem is reached
  - associated to performance & robustness

→ Test driven development when needed
Version Control System

• RhodeCode: manage repositories "@HOME", with Git or Hg, pull request, forks, gists

• Git: allow diff, reset, blame, diff, stash

• Workflow: uses local branches, repositories like upstream, origin, ...

• pull request: pieces of codes are integrated after peer review

→ Agile method, extreme coding...
Continuous Integration

After a commit from contributor:

- clang-analyze & cppcheck
- coding style & documentation
- unit & integration tests
- compilers & libraries (portability)
- push on official repository

→ using a local "TeamCity"
A library for AMR: SAMRAI

Developed at LLNL, manage plenty of nice features:

"patch based", dynamic mesh refinement, user defined data, load balancing, interface to solver libraries, visualization support,... & open source!
splitting method for particles: [Smets et al. 2018]

→ exact solution... eventually expensive

→ approximate solution results from optimisation
Concluding remarks

• We need software engineering
• We need collaborations
• We need open source codes
• We need engineer in HPC
• We need money

→ But we already have computers...
For each new PhD student...
About the Hall effects

- (Hall) $E_{XY}$ electric field associated to $J_Z$ and $B_{XY}$
- $J_Z$ grows at the tip of each loops when colliding
  → quadrupolar $B_Z$ grows because $E_{XY}$ is no more curl-free
- $J_{XY}$ associated to this out-of-plane magnetic field
  → Carried by electrons (protons are demagnetized)
Reconnection Rate

Slope of the reconnected flux: \( E_Z = -\partial_t A_Z \)

Reach the “holly” value of 0.2...

→ The outflow speed is around 0.2 times the (upstream) Alfvén speed (not yet normalized)
Out-of-plane quadrupolar (Hall) Magnetic Field

Its value clearly increases prior the reconnection onset
→ Can not be a consequence of the reconnection process
Double hump structure like the one of the $E_Z$ component
→ Close connection between these two components

Ultra-relativistic pair-plasmas ($\gamma \sim 10^3$, $\sigma \sim 10^4$) (collisionless) Shock-driven reconnection
→ EM energy to synchrotron emitting electrons ($\chi$ & $\gamma$)
Accretion disks [Gouveia dal Pino & Lazarian, 2005]

Can explain the steep power-law state of photons for $\beta \leq 1$

→ Could be extended to AGNs & YSOs,
\(\gamma\) ray bursts (Fireball model) [Thompson, 1994]

Ultra-relativistic with \(\beta \leq 10^{-4} \rightarrow f(\gamma) \propto \gamma^{-p}\) with \(p \sim -2.2\) → Associated \(p \sim -1.6\) for synchrotron photons