Rotating stars in two dimensions

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Outline

1. Some background
2. The brief history of 2D models
3. The ideal model
4. Numerics
5. In practice - first results
   - The velocity
   - The virial
   - Some examples of models
   - Comparisons to observations
   - Gravity darkening

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Some astrophysical motivations

All the stars rotate, and young stars are fast...

We wish to understand

1. the structure, the flows and the atmosphere of a fast rotating star with a given (initial) chemical composition
2. the consequences of fast rotation on the eigenspectrum of such a star
3. the way these stars lose angular momentum and what are the consequences
4. the evolution of rotation during the lifetime of the stars
5. the consequences of rotation on abundances
6. the relations with magnetic activity
7. the validity of 1D models on the rotation axis

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To summarize :

Build self-consistent models to monitor all secular effects of rotation on stars.

A timely question

Optical or IR interferometry

VLTI (ESO), NPOI (USA), CHARA (USA) allow a gross imaging of stars. Examples :

A timely question
Interferometry

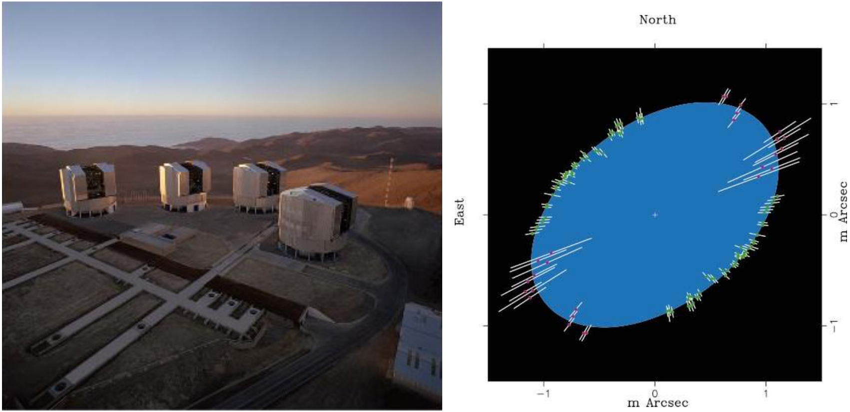

FIGURE – Achernar seen with VLTI (Domiciano de Souza et al. AA, 2003)

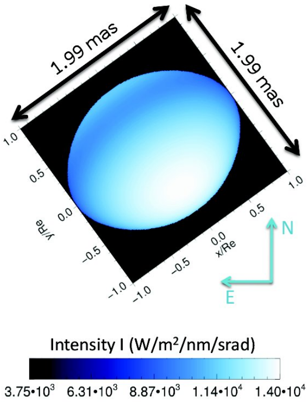

FIGURE – Achernar with VLTI (Domiciano de Souza et al. 2014, AA **569**)

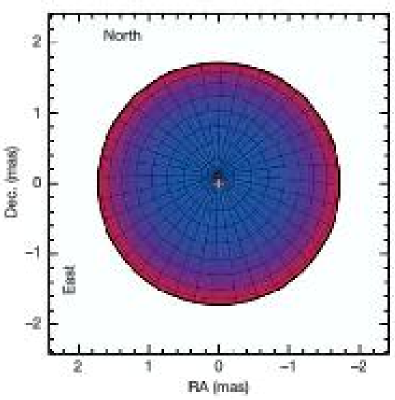

FIGURE – Vega viewed with NPOI (Peterson et al. ApJ 2006), $\Omega \sim 0.93\Omega_B$

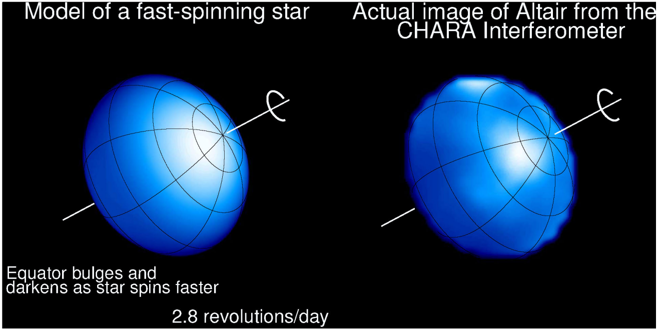

FIGURE – Altair viewed with CHARA (Monnier et al. 2007).

plus many diameters of A-type stars like

- α PsA (with a dusty debris disc, planet ?), β Leo (a δ-Scuti star), β Pic (very young with planets),

  ...

a growing series.

- Altair (flatness $\sim$ 20%) is a δ-Scuti (Buzasi et al. 2005) just as Ras Alhague (α Oph)
- CoRoT/KEPLER/WIRE/MOST : They yield a large set of oscillating fast rotating stars.

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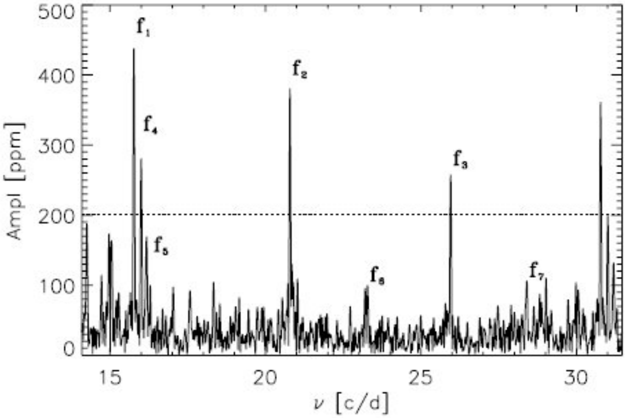

FIGURE – Part of the oscillation spectrum of Altair from WIRE (Buzasi et al. 2005).

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The historical steps of 2D-models

- The pioneers : James (1964) and Roxburgh, Griffith & Sweet (1965)
- The American series : Bodenheimer, Jackson, Mark & Ostriker (1968-1973)
- The Canadian series : Clement (1974-1994)
- The Japanese school : Eriguchi (1978-1997)
- The German-Japanese school : Eriguchi-Müller (1985-1993)
- The revival : Roxburgh 2004, Jackson et al. 2005 et Deupree 2011
- The French-Spanish series : Rieutord & Espinosa Lara (2005 - ...)

The American way

- Mark & Ostriker 1968 : *Rapidly rotating stars. I. The self-consistent-field method* :
  The Poisson equation for the potential is solved using the Green integral :

  $$\phi = -G \int \frac{\rho}{|\vec{r} - \vec{r}'|} d^3\vec{r}'$$

  so that boundary conditions on ϕ are readily met.

Models are quite simple (polytropic eos) but face numerous problems :

- the code was not flexible,
- the code did not work with $M \leq 9\ M_\odot$,
- the code could not deal with very fast rotation, large density contrasts.

Other attempts
1973-1997

- Numerical difficulties were plaguing the attempts : solutions were not precise : virial test $=2\,10^{-4}$ (Clement 1973), $=4\,10^{-4}$ (Eriguchi & Müller 1985).
- Differential rotation was imposed
- Physics was usually very simplified

Some exotic configurations

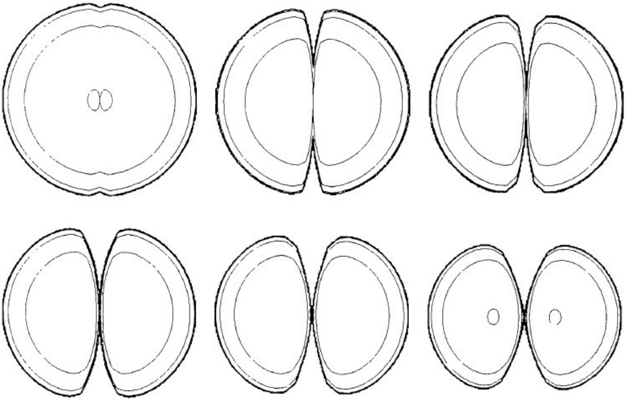

FIGURE – Polytropes with $\Omega(s) = \Omega_0/(1 + s^2/A^2)$.

The revival

- Roxburgh 2004, 2006 : hydrostatic models with ad hoc differential rotation for seismology : but no follow up.
- Jackson, McGregor & Skummanich 2004, 2005 (ApJ, ApJS) : try to model the results of interferometry on Achernar, but hydrostatic + adhoc DR.
- Deupree 2011, ApJ, similar models as above but focus on an A-star (α-Oph). First, prediction of the SED + tentative predictions on the eigenspectrum.
- Rieutord & Espinosa Lara (2005–) head into the dynamics...

Conclusions

- Previous attempts show that the problem is tough : Robust numerics is desired.
- Differential rotation and meridional circulation need to be included at the outset.
- We should reach the state “stellar evolution”

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The model should describe an isolated, non-magnetic star, in a steady state or quasi-steady state.

$$\begin{cases} \Delta\phi = 4\pi G\rho \\ \rho T\vec{v}\cdot\vec{\nabla}S = -\mathrm{Div}\vec{F} + \varepsilon_* \\ \rho(2\vec{\Omega}_* \wedge \vec{v} + \vec{v}\cdot\vec{\nabla}\vec{v}) = -\vec{\nabla}P - \rho\vec{\nabla}(\phi - \frac{1}{2}\Omega_*^2 s^2) + \vec{F}_v \\ \mathrm{Div}(\rho\vec{v}) = 0. \end{cases} \tag{1}$$

≡ the equations of a steady flow of a compressible, self-gravitating fluid, with nuclear reactions...

Energy flux

$$\vec{F} = -\chi_r \vec{\nabla} T - \frac{\chi_{\rm turb} T}{\mathcal{R}_M} \vec{\nabla} S$$

Viscous force

$$\vec{F}_v = \mu \vec{\mathcal{F}}_\mu(\vec{v}) = \mu \Big[\Delta\vec{v} + \frac{1}{3}\vec{\nabla}\left(\vec{\nabla}\cdot\vec{v}\right) + 2\left(\vec{\nabla}\ln\mu\cdot\vec{\nabla}\right)\vec{v} + \vec{\nabla}\ln\mu \times (\vec{\nabla}\times\vec{v}) - \frac{2}{3}\left(\vec{\nabla}\cdot\vec{v}\right)\vec{\nabla}\ln\mu \Big] .$$

or the prescription of the Reynolds stress tensor.

$$\begin{cases} P \equiv P(\rho, T) \quad \text{OPAL} \\ \kappa \equiv \kappa(\rho, T) \quad \text{OPAL} \\ \varepsilon_* \equiv \varepsilon_*(\rho, T) \quad \text{NACRE} \end{cases} \tag{2}$$

- On pressure

$$P_s = \frac{2}{3}\frac{\overline{g}}{\overline{\kappa}}$$

- On velocity

$$\vec{v}\cdot\vec{n} = 0 \quad \text{and} \quad ([\sigma]\vec{n})\wedge\vec{n} = \vec{0}$$

- On temperature

$$\vec{n}\cdot\vec{\nabla}T + T/L_T = 0$$

The total angular momentum

$$\int_{(V)} r \sin\theta \rho u_\varphi \, dV = L$$

or the equatorial velocity

$$v_\varphi(r = R, \theta = \pi/2) = V_{\mathrm{Eq}}$$

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The numerical side

- Present evolution codes use less than 10^4 grid points
- To keep similar performance a 2D grid should not exceed $10^2 \times 10^2$ suggesting the use of spectral methods.

Method

- The shape of the star is unkown and should be derived,
- On this surface boundary conditions apply.

Coordinates should be adapted to the geometry of the star.

Mappings

Method : taken from Bonazzola, Gourgoulhon et Marck (1998) :
A mapping transforms the natural coordinates ζ, θ, φ into the spherical coordinates (same topology) :

$$r = \zeta + A(\zeta)(R(\theta) - 1), \quad \theta' = \theta, \quad \varphi' = \varphi \tag{3}$$

where $A(\zeta)$ is a polynomial chosen by the user.

Mappings

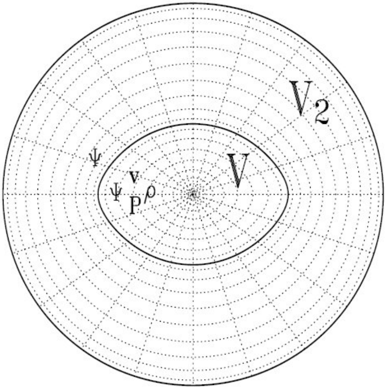

FIGURE – The mapping.

Mappings

The natural coordinates are curvilinear and non-orthogonal. The metric is

$$g^{\zeta\zeta} = \frac{r^2 + r_\theta^2}{r^2 r_\zeta^2}, \qquad g^{\zeta\theta} = -\frac{r_\theta}{r^2 r_\zeta},$$

$$g^{\theta\theta} = \frac{1}{r^2}, \qquad g^{\varphi\varphi} = \frac{1}{r^2 \sin^2\theta}$$

Discretisation

Spectral : Chebyshev “radially” and spherical harmonics horizontally :

$$\phi = \sum_{\ell=0,2,\ldots} \phi_\ell(\zeta_i) Y_\ell^m$$

Spectral space horizontally, collocation points radially.

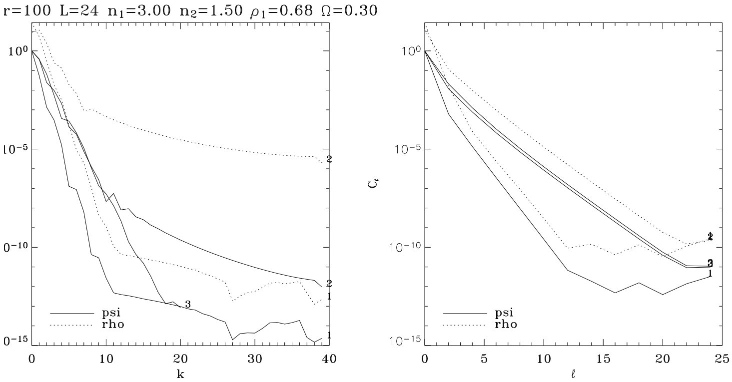

FIGURE – Spectra of the solutions.

The algorithm for iterations

How should we move from an approximate solution $\vec{X}_N$ to a better solution $\vec{X}_{N+1}$?

- The fixed point or Picard's algorithm
- Newton-Raphson algorithm

Example :

$$\Delta\phi = \mathrm{RHS}(\phi)$$

To be solved in spheroidal coordinates

$$\tilde{\Delta}\Phi_{N+1} = \frac{1}{g}(\mathrm{NS}+\mathrm{RHS})_N + \left(1-\frac{g^{\zeta\zeta}}{g}\right)\left(\lambda(\tilde{\Delta}\Phi)_N + (1-\lambda)(\tilde{\Delta}\Phi)_{N-1}\right)$$

λ relaxation parameter. Pro : easy to implement Cons : slow convergence.

Newton's algorithm

$$\vec{F}(\vec{x}) = \vec{0},$$

$$\delta\vec{F}(\vec{x}) = \mathbf{J}(\vec{x})\delta\vec{x}.$$

where $\mathbf{J}$ is the jacobian matrix of the nonlinear system.

$$\mathbf{J}(\vec{x}^N)\delta\vec{x}^N = -\vec{F}(\vec{x}^N)$$

and $\vec{x}^{N+1} = \vec{x}^N + \delta\vec{x}^N$, with a judicious choice of $\vec{x}^0$,

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The problem of the velocity field

Velocity fields are solutions of

$$\begin{cases} \rho\vec{v}\cdot\vec{\nabla}\vec{v} = -\vec{\nabla}P - \rho\vec{\nabla}\phi + \vec{F}_v \\ \\ \mathrm{Div}(\rho\vec{v}) = 0 \end{cases} \qquad (4)$$

Difficulties :

- ρ varies over 10 orders of magnitude : $\rho_c/\rho_s \sim 10^{10}$
- Viscosity generate extremely small scales $L \lesssim 10^{-4}R_*$.

The problem of the velocity field

Solution (at the moment) :

- The star sliced into multidomains to deal with large variations of density (spectral elements).
- The boundary condition on velocity is changed so as to account for Ekman layers without computing them.

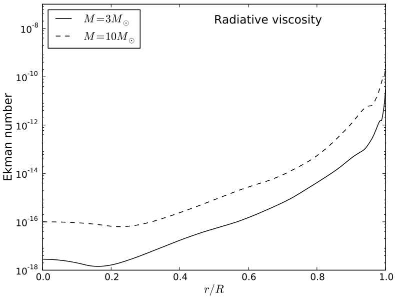
About viscosity in early-type stars

- At microscopic level

$$\mu_{\rm rad} = \frac{2aT^4}{15c\kappa\rho}$$

- Turbulent viscosity (Zahn, 1992)

$$\mu_{\rm turb} = \rho\frac{{\rm Ri}_c K}{3}\left(\frac{s}{\mathcal{N}}\frac{{\rm d}\Omega}{{\rm d}s}\right)^2$$



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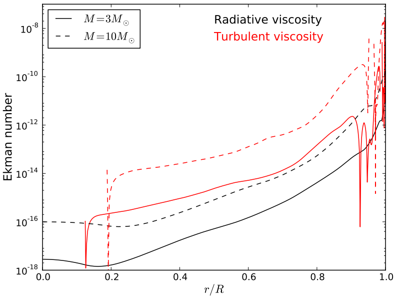


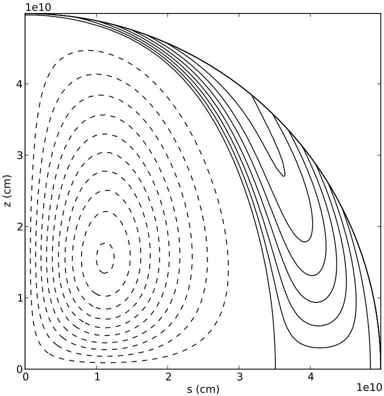
The Ekman number measures the importance of viscosity

$$E = \frac{\nu}{2\Omega R^2} \ll 1$$

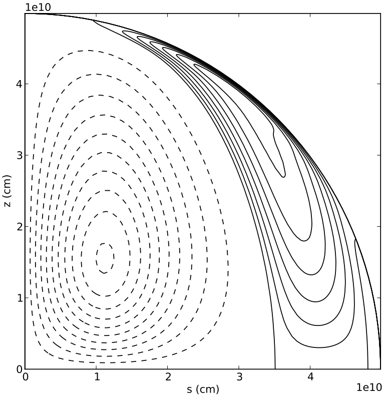
implies

- Numerical difficulties for solutions with viscosity
- Using asymptotic methods is possible to eliminate Ekman layers

Meridional circulation

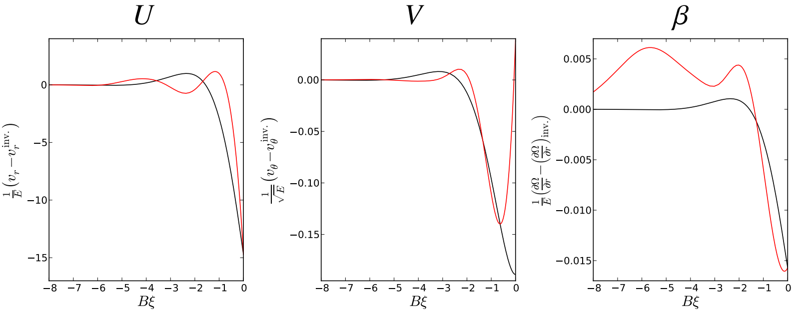


Asymptotic solution



Full solution $(E = 10^{-7})$

Boundary layer corrections

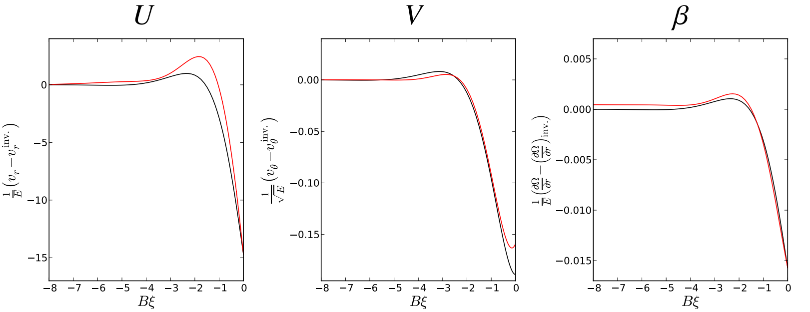


— Asymptotic solution

— Full solution

$E = 10^{-5}$

Boundary layer corrections

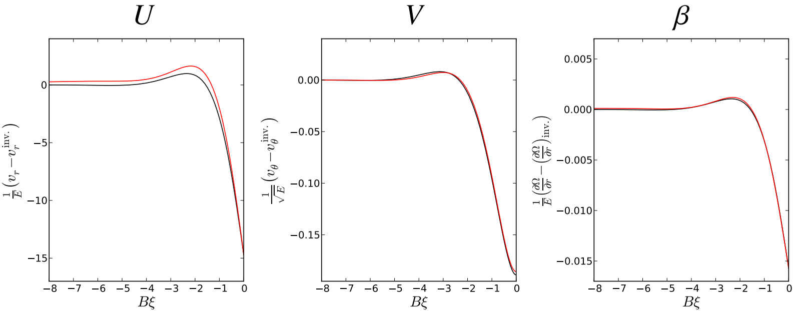


— Asymptotic solution

— Full solution

$E = 10^{-6}$

Boundary layer corrections

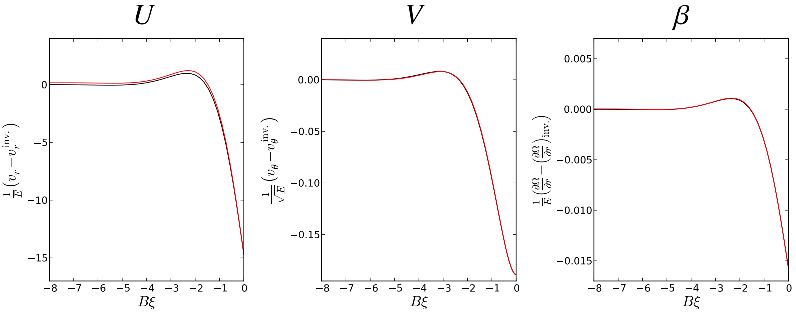


— Asymptotic solution

— Full solution

$E = 10^{-7}$

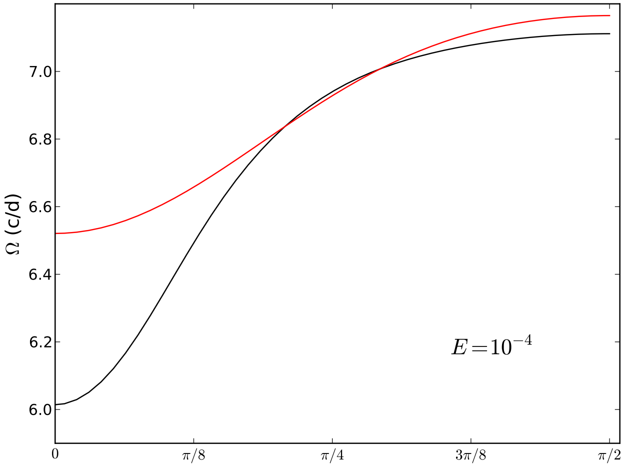
Boundary layer corrections



— Asymptotic solution
— Full solution

$E = 10^{-8}$

Surface differential rotation



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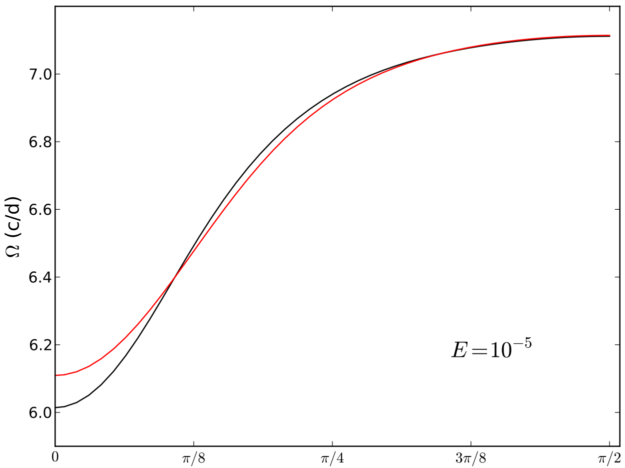
Surface differential rotation



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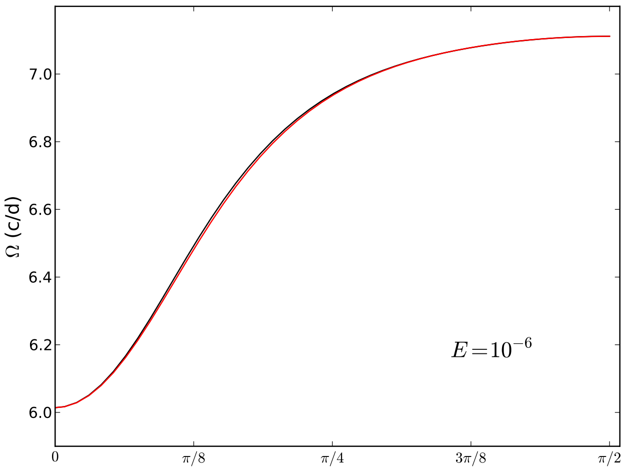

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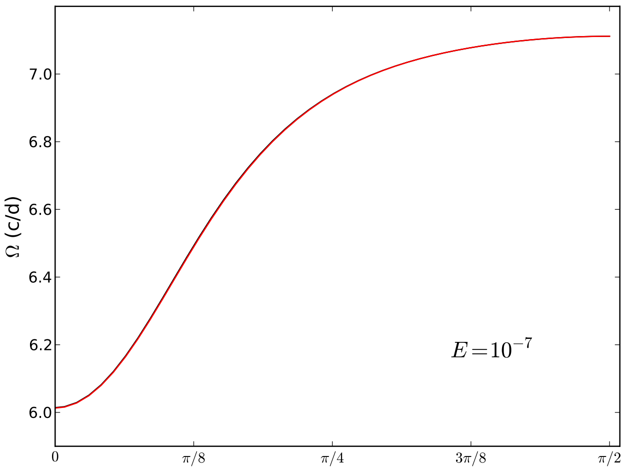
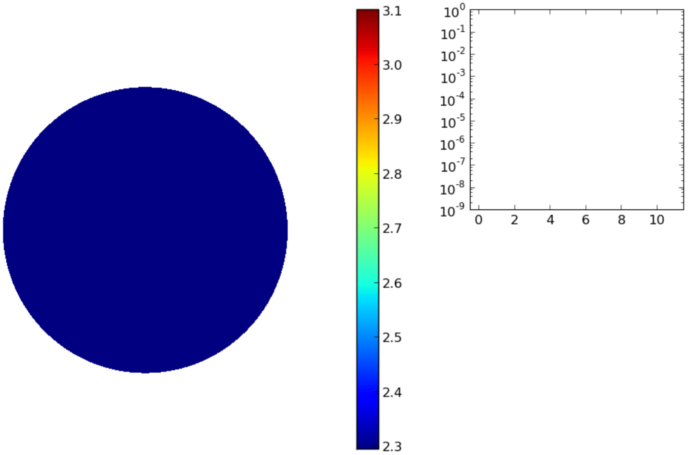


— Asymptotic solution

— Full solution

Convergence towards a solution



Internal consistency of the solutions

Consistency/internal precision is monitored through 4 criteria :

- Truncation error (from spectra, Chebyshev or SH)
- Round-off error (sensitivity to initial conditions)
- The virial
- The energy balance

The virial test

$$\int_{(V)} \vec{r}\cdot\left[2\vec{\Omega}\wedge\rho\vec{u} + \rho\vec{u}\cdot\vec{\nabla}\vec{u} + \rho\vec{\nabla}\phi - \rho\Omega^2 s\vec{e}_s - \mathbf{Div}[\sigma]\right] dV = 0$$

This is usually normalized by potential energy :

$$W = \frac{1}{2}\int_{(V)} \rho\phi\, d^3\vec{r}$$

Example of some results

TABLE – Fundamental parameters of rotating $3M_\odot$ stellar models.

Ω/Ω_K^b	$R(R_\odot)$	ε	v_{eq}(km/s)	$L(L_\odot)$	$T_{eff}(10^3K)$	$\log g_e$
0.0	1.97	0.00	0.0	81.2	12.36	4.33
0.3	1.96(p) 2.05(e)	0.04	158.6	80.0	12.50(p) 11.97(e)	4.33(p) 4.25(e)
0.5	1.95(p) 2.19(e)	0.11	255.5	78.4	12.69(p) 11.31(e)	4.34(p) 4.11(e)
0.9	1.93(p) 2.74(e)	0.29	411.5	76.4	12.92(p) 8.91(e)	4.34(p) 3.32(e)

TABLE – Fundamental parameters of rotating $3M_{\odot}$ stellar models.

Ω/Ω_K	ρ_c(cgs)	$T_c(10^7$K)	ρ_s/ρ_c	Virial test	Energy test
0.0	40.8	2.43	$2.0 \cdot 10^{-11}$	$5.1 \cdot 10^{-11}$	$5.8 \cdot 10^{-7}$
0.3	41.0	2.43	$2.1 \cdot 10^{-11}$	$8.0 \cdot 10^{-11}$	$3.4 \cdot 10^{-7}$
0.5	41.3	2.42	$2.1 \cdot 10^{-11}$	$2.0 \cdot 10^{-10}$	$6.1 \cdot 10^{-6}$
0.9	41.6	2.42	$1.9 \cdot 10^{-11}$	$2.3 \cdot 10^{-10}$	$1.1 \cdot 10^{-5}$

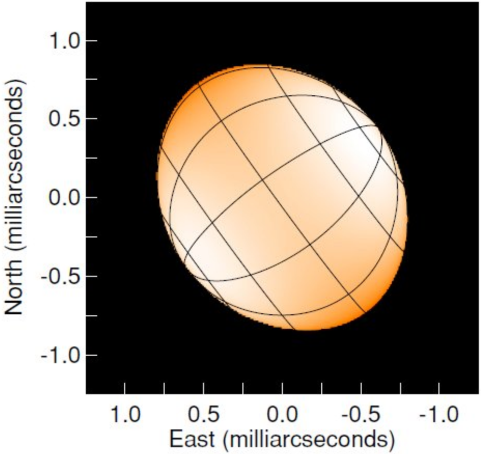
Comparison to 1D models

Mass	$\delta R/R$	$\delta L/L$	$\delta\rho_c/\rho_c$	$\delta T_c/T_c$
3	10^{-3}	3×10^{-3}	5×10^{-3}	8×10^{-3}
7	6×10^{-3}	3×10^{-2}	5×10^{-2}	3×10^{-4}

TABLE – Comparison of the results between the one-dimensional of the ESTER code and the one-dimensional code TGEC. Same results with CESAM2k

Ras Alhague, α Ophiuchi

Reconstruction of an image with the optical interferometer CHARA.



Ras Alhague - values

	Observations	Model
R_{eq}	2.858±0.015	2.859
R_{pol}	2.388±0.013	2.379
T_{eq}	7570±124 K	7825 K
T_{pol}	9384±154 K	9333 K
$L/L_\odot$	31.3±1	32.8
V_{eq}	240±12 km/s	244 km/s

TABLE – Adjusted parameters : $M=2.25M_\odot$, $\omega_k = 0.63$, $X_c = 0.27$

Regulus, α Leonis

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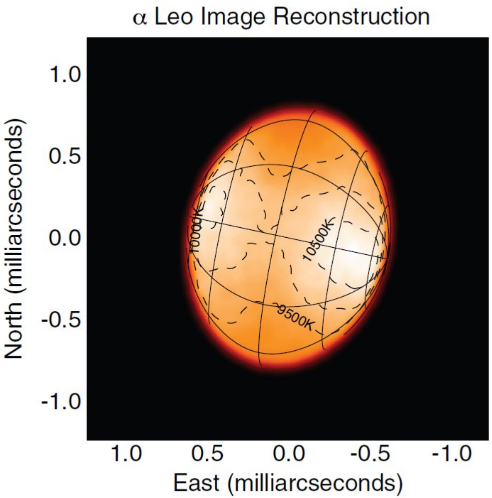
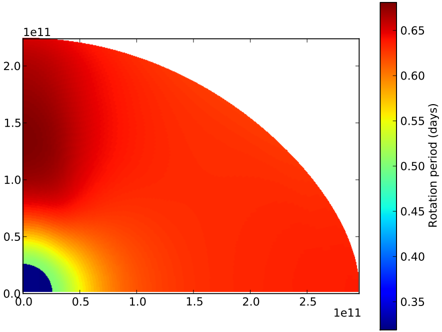
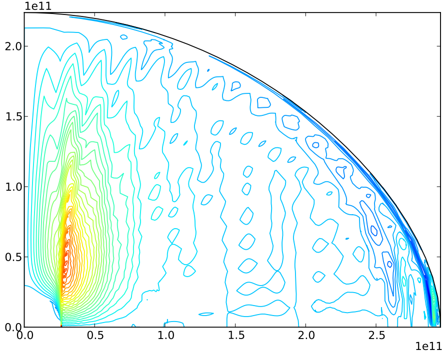


Regulus (α Leo)



	Model values	Measured values (Che et al 2011)
M ($M_\odot$)	4.07	4.15±0.06
R_{eq} ($R_\odot$)	4.254	4.21±0.07
R_{pol} ($R_\odot$)	3.22	3.22±0.05
L ($L_\odot$)	340.5	341±27
v_{eq} (km/s)	337.61	336±24
T_{eq} (K)	11038	11010±520
T_{pol} (K)	14495	14520±690
$X_{core}/X_{env.}$	0.5	
Virial test	$5.6 \cdot 10^{-10}$	
Energy test	$2.1 \cdot 10^{-5}$	

Regulus (α Leo)



	Model values	Measured values (Che et al 2011)
M ($M_\odot$)	4.07	4.15±0.06
R_{eq} ($R_\odot$)	4.254	4.21±0.07
R_{pol} ($R_\odot$)	3.22	3.22±0.05
L ($L_\odot$)	340.5	341±27
v_{eq} (km/s)	337.61	336±24
T_{eq} (K)	11038	11010±520
T_{pol} (K)	14495	14520±690
$X_{core}/X_{env.}$	0.5	
Virial test	$5.6{\cdot}10^{-10}$	
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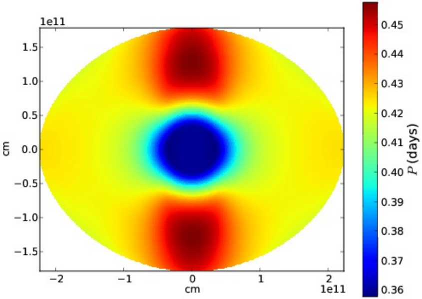


FIGURE – Differential rotation of a $5M_{\odot}$ star with $\Omega = 0.7\Omega_k$.

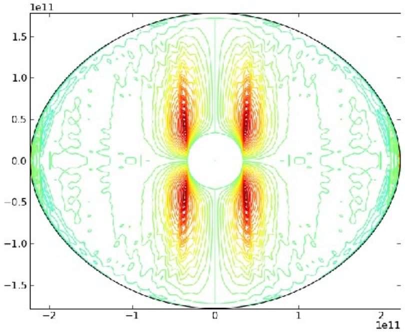
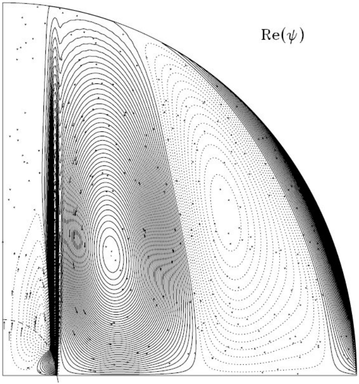


FIGURE – Meridional circulation of a $5M_\odot$ star with $\Omega = 0.7\Omega_k$.

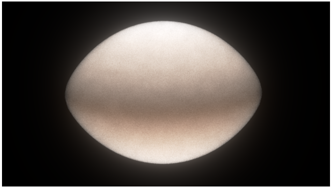
... driven by a viscosity and density jump at the core-envelope interface.



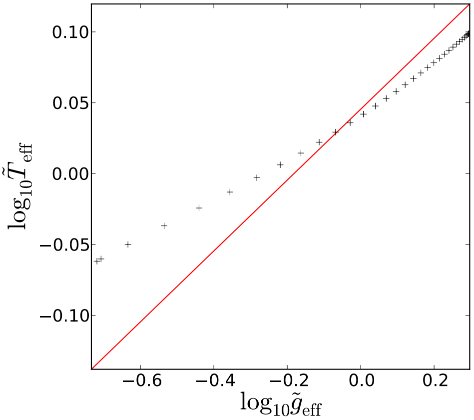
Gravity darkening

- The effective temperature of rotating stars is not uniform across its surface.
- Poles are hotter and brighter than the equator.
- Von Zeipel's law (Barotropic model, 1924) :

$$T_{\rm eff} \propto g_{\rm eff}^{1/4}$$



Gravity darkening



$\Omega = 0.9\Omega_k$

— : Von Zeipel's law

++ : ESTER model ($M = 3M_\odot$)

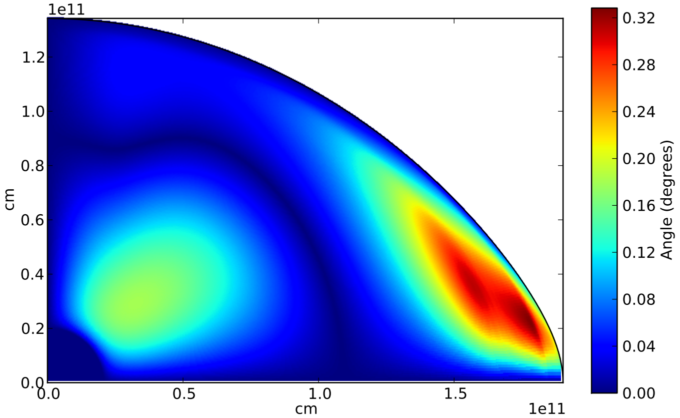
Hypothesis

- Deviation from barotropicity is small.
- Energy flux is antiparallel to the local effective gravity.

$$\mathbf{F} = -f(r, \theta)\mathbf{g}_{\text{eff}}$$

- Convection : Energy transport driven by bouyancy.
- Radiation : Angle between ∇T and $\mathbf{g}_{\text{eff}}$ remains small ($< 1^\circ$).

Angle between ∇T and ∇p $(\Omega = 0.9\Omega_k)$



A new model for gravity darkening

In the envelope of a star, where no heat is generated :

$$\nabla \cdot \mathbf{F} = 0 \qquad \Longrightarrow \qquad \mathbf{g}_{\rm eff} \cdot \nabla f + f \nabla \cdot \mathbf{g}_{\rm eff} = 0$$

Energy flux depends only on the shape of the equipotential surfaces and hence on mass distribution.

Rapidly rotating stars are usually intermediate or high mass stars, and thus centrally condensed.
For simplicity we use $\mathbf{g}_{\rm eff}$ given by the **Roche model**.

$$\mathbf{g}_{\rm eff} = -\frac{GM}{r^2}\mathbf{e}_r + \Omega^2 r \sin\theta \mathbf{e}_s$$

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A new model for gravity darkening

$$T_{\text{eff}} = \left(\frac{L}{4\pi\sigma GM}\right)^{1/4} \sqrt{\frac{\tan\theta_0}{\tan\theta}}\, g_{\text{eff}}^{1/4}$$

where

$$\cos\theta_0 + \ln\tan\frac{\theta_0}{2} = \frac{1}{3}\omega^2\tilde{r}^3\cos^3\theta + \cos\theta + \ln\tan\frac{\theta}{2}$$

- Gravity darkening depends only on $\omega = \frac{\Omega}{\Omega_k}$. $\left(\Omega_k = \sqrt{\frac{GM}{R_e^3}}\right)$
- For slow rotation $\theta_0 \approx \theta$ and we recover von Zeipel's law.

Gravity darkening exponent : $T_{\rm eff} \propto g_{\rm eff}^{\beta}$

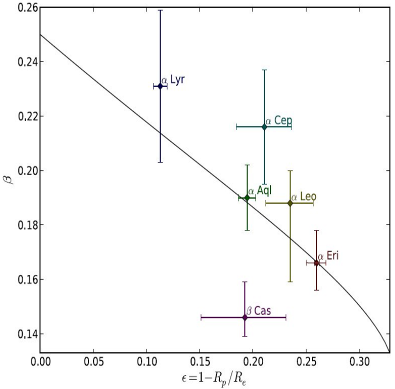
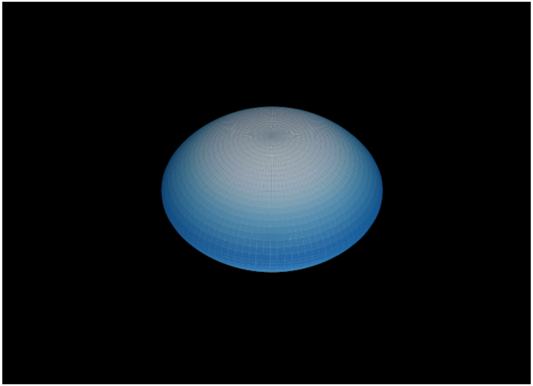


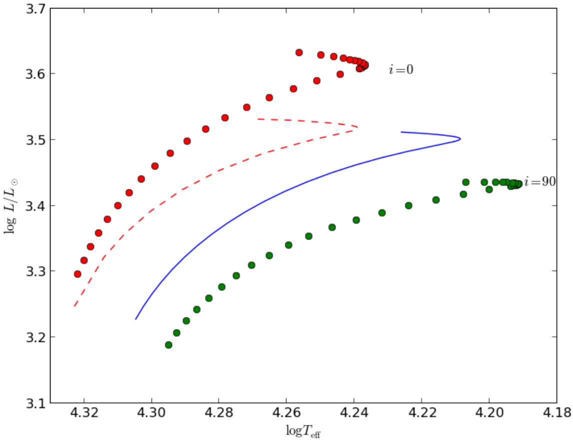
FIGURE – Observed values of β and a simple model of Espinosa Lara & Rieutord (2011).

Gravity darkening of Achernar (α Eri)

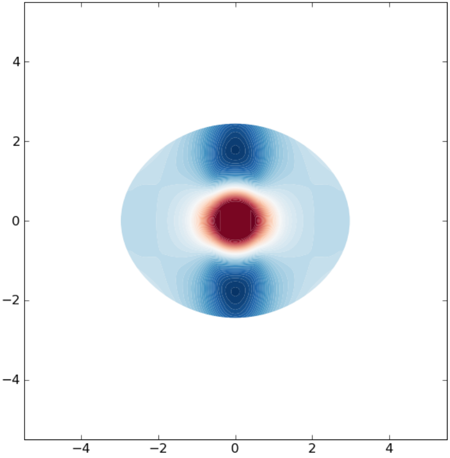


Towards evolution

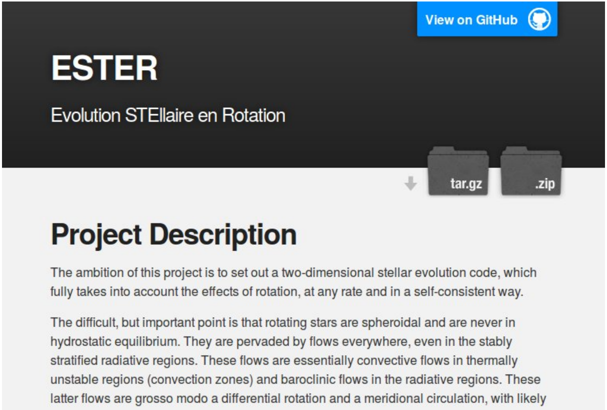
HR diagram track of a $7M_\odot$ star of constant angular momentum, starting at $\Omega/\Omega_k = 0.5$.



Evolution of a $5M_\odot$ star on the main sequence



ESTER : The Code



View on GitHub

ESTER

Evolution STEllaire en Rotation

tar.gz .zip

Project Description

The ambition of this project is to set out a two-dimensional stellar evolution code, which fully takes into account the effects of rotation, at any rate and in a self-consistent way.

The difficult, but important point is that rotating stars are spheroidal and are never in hydrostatic equilibrium. They are pervaded by flows everywhere, even in the stably stratified radiative regions. These flows are essentially convective flows in thermally unstable regions (convection zones) and baroclinic flows in the radiative regions. These latter flows are grosso modo a differential rotation and a meridional circulation, with likely

FIGURE – Freely available on the www

Outlooks

- Extend to lower masses
- take into account anisotropic mass loss (hence aml) : cf poster of **Damien Gagnier**
- implement the nuclear clock

Outlooks

But presently we can

- do asteroseismology of MS stars at any rotation rate
- invert interferometric visibilities and closure phases
- determine the validity of 1D models
- monitor evolution on the MS at constant angular momentum
- ...

First steps on real evolution

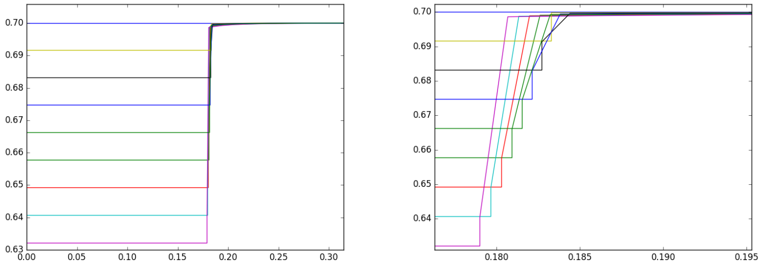


FIGURE – Hydrogen mass fraction X in a convective core for a 5 $\mathrm{M}_{\odot}$ star. $\Delta t = 2$ Myrs.

Some references

- Rieutord, Espinosa Lara & Putigny (2016), J. Comput. Phys. 318, 277
- Espinosa Lara & Rieutord (2013), A&A,**552**, A35
- ESTER website : http ://ester-project.github.io/ester/