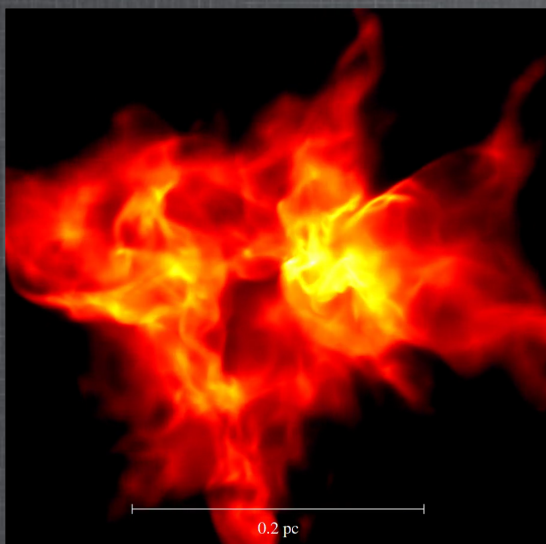
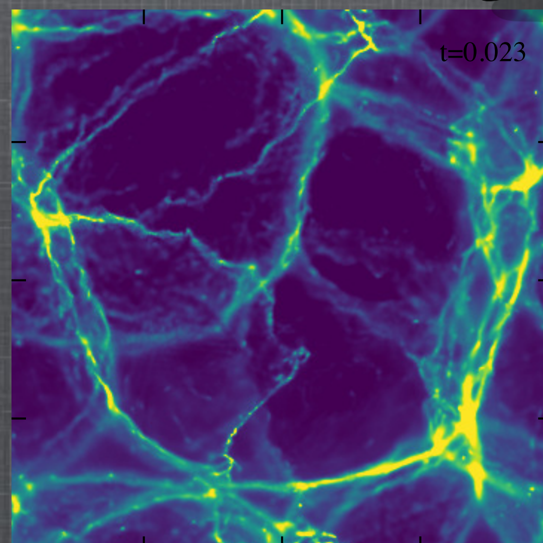
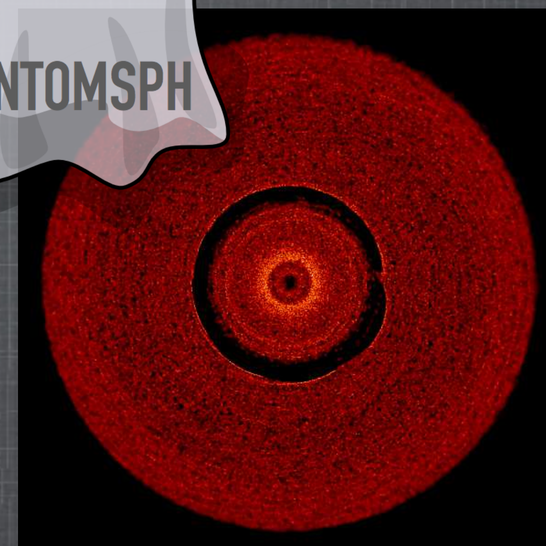
**Lyon - 03 / 07 / 2017**

# *Astrophysics with the SPH code PHANTOM*



A code from *D.J. Price*

presented by G. Laibe

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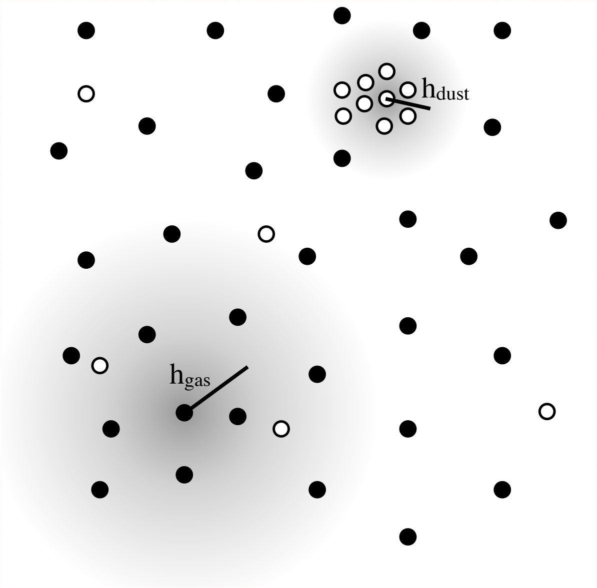
# A short note about *SPH*




Smoothed
Particles
Hydrodynamics

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\rho(\nabla \cdot \boldsymbol{v}),$$

$$\rho_a = \sum_b m_b W(|\boldsymbol{r}_a - \boldsymbol{r}_b|, h_a) \qquad h_a = h_{\mathrm{fact}} n_a^{-1/3} = h_{\mathrm{fact}} \left(\frac{m_a}{\rho_a}\right)^{1/3}$$

$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = -\frac{\nabla P}{\rho} + \Pi_{\mathrm{shock}} + \boldsymbol{a}_{\mathrm{ext}}(\boldsymbol{r}, t) + \boldsymbol{a}_{\mathrm{sink-gas}} + \boldsymbol{a}_{\mathrm{selfgrav}},$$

$$\frac{\mathrm{d}\boldsymbol{v}_a}{\mathrm{d}t} = -\sum_b m_b \left[\frac{P_a + q_{ab}^a}{\rho_a^2 \Omega_a} \nabla_a W_{ab}(h_a) + \frac{P_b + q_{ab}^b}{\rho_b^2 \Omega_b} \nabla_a W_{ab}(h_b)\right] + \boldsymbol{a}_{\mathrm{ext}}(\boldsymbol{x}_a, t) + \boldsymbol{a}_{\mathrm{sink-gas}}^a + \boldsymbol{a}_{\mathrm{selfgrav}}^a$$

$$\frac{\mathrm{d}u}{\mathrm{d}t} = -\frac{P}{\rho}(\nabla \cdot \boldsymbol{v}) + \Lambda_{\mathrm{shock}} - \frac{\Lambda_{\mathrm{cool}}}{\rho}$$

$$\frac{\mathrm{d}u_a}{\mathrm{d}t} = \frac{P_a}{\rho_a^2 \Omega_a} \sum_b m_b \boldsymbol{v}_{ab} \cdot \nabla_a W_{ab}(h_a) + \Lambda_{\mathrm{shock}} - \frac{\Lambda_{\mathrm{cool}}}{\rho}$$

# Code genesis

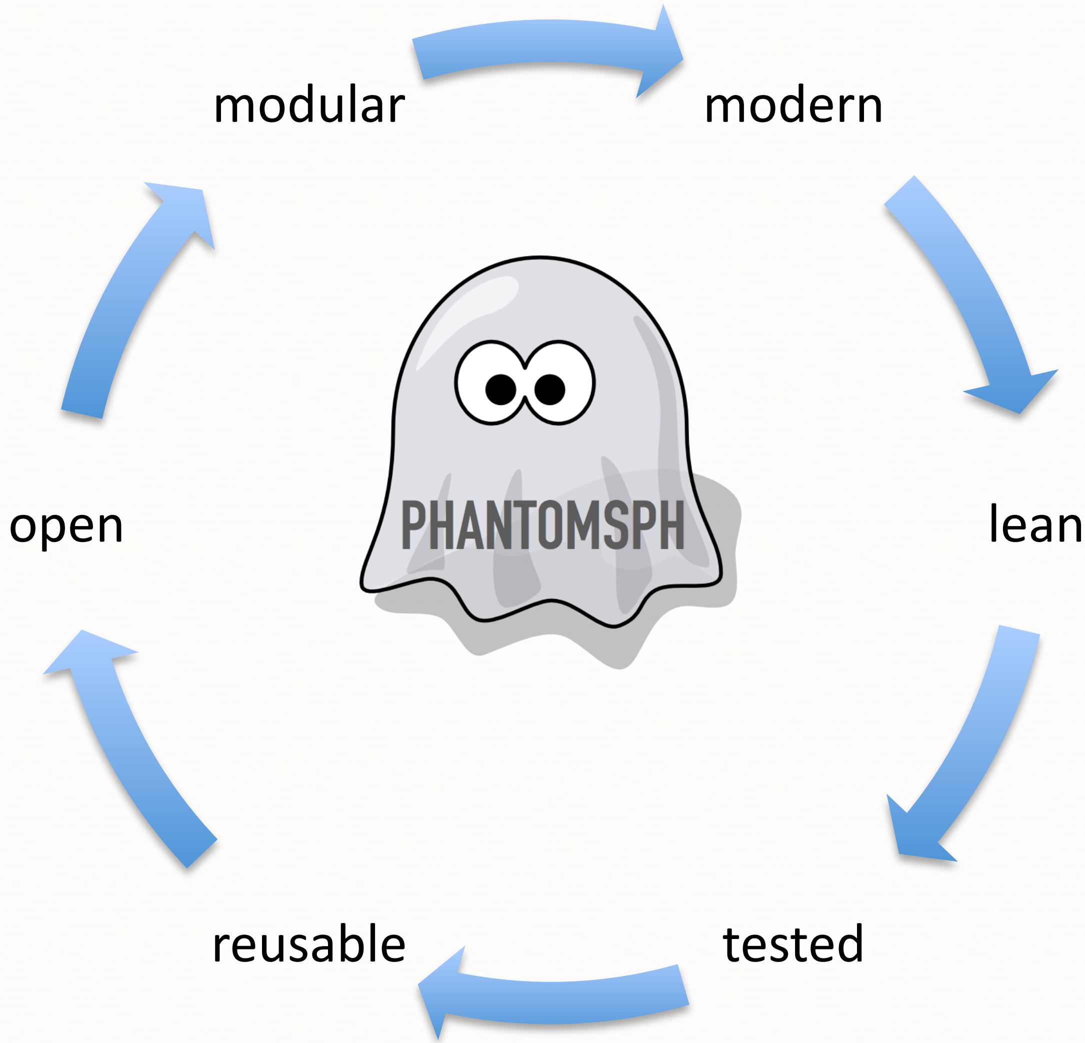
SPH useful for:
- free boundaries
- complex geometry
- resolution in mass
- advected bodies


But:

| Code | Who | Limitations |
|---|---|---|
| Gadget 2 | Springel | Old hydro solver |
| Gadget 3 | Springel | Private |
| Gasoline 2 | Wadsley | no MHD, no dust |
| Seren | Hubber | no MHD, no dust |
| SPHNG | Bate | Private |

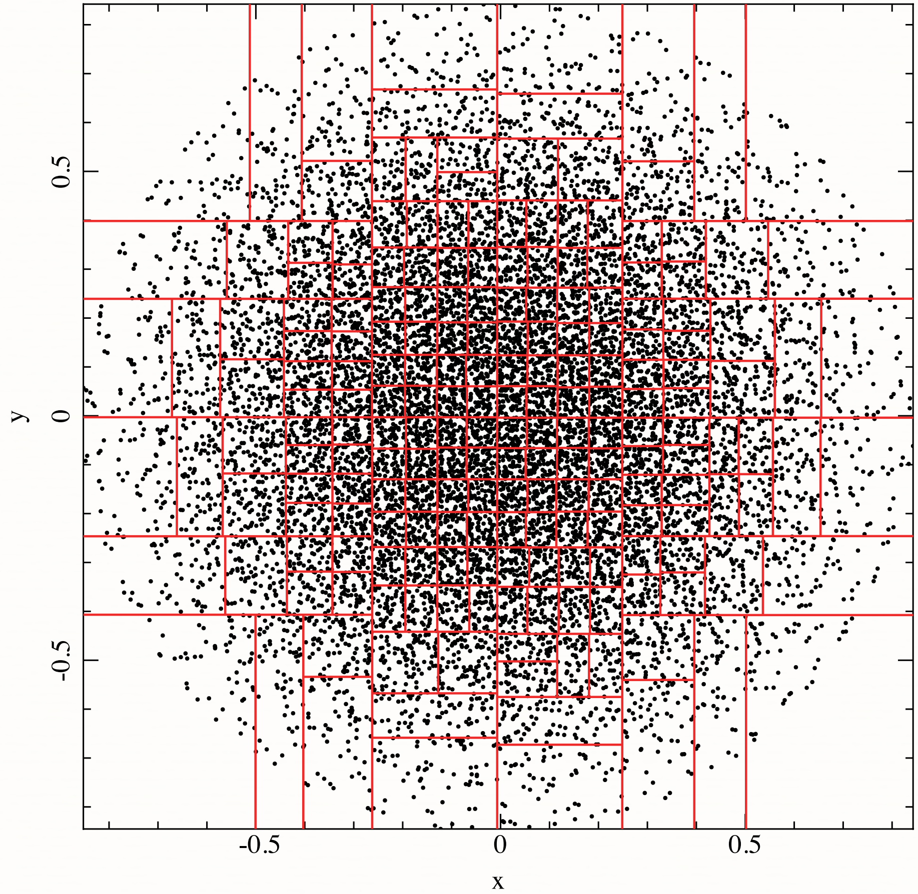

Price et al. (2018)

# Code philosophy



# Neighbours findings

Fast, low-memory, parallel (OPEN MP/MPI) for neighbour findings




PHANTOM KDtree

# Discontinuities

Discontinuity term:

$$\Lambda_{\rm shock} \equiv -\frac{1}{\Omega_a \rho_a} \sum_b m_b v_{{\rm sig},a} \frac{1}{2} (\boldsymbol{v}_{ab} \cdot \hat{\boldsymbol{r}}_{ab})^2 F_{ab}(h_a)$$

$$+ \sum_b m_b \alpha_u v^u_{\rm sig} (u_a - u_b) \frac{1}{2} \left[ \frac{F_{ab}(h_a)}{\Omega_a \rho_a} + \frac{F_{ab}(h_b)}{\Omega_b \rho_b} \right]$$

$$+ \Lambda_{\rm artres}, \qquad (42)$$

→ artificial viscosity

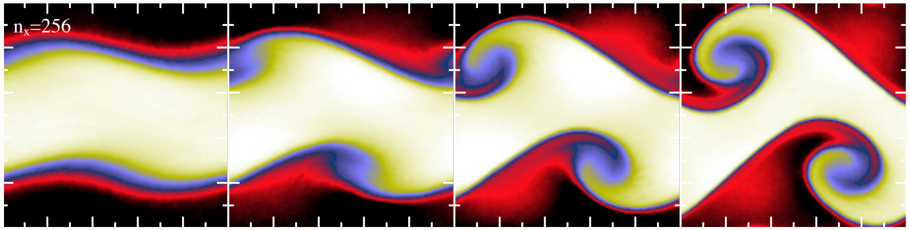
→ artificial conductivity

→ artificial resistivity

Signal velocity:

$$v^u_{\rm sig} = \sqrt{\frac{|P_a - P_b|}{\overline{\rho}_{ab}}}$$ no self-gravity

$$v^u_{\rm sig} = |\boldsymbol{v}_{ab} \cdot \hat{\boldsymbol{r}}_{ab}|$$ with self-gravity




Shock capturing

$$\frac{{\rm d}\alpha_a}{{\rm d}t} = -\frac{(\alpha_a - \alpha_{{\rm loc},a})}{\tau_a}$$

$$\alpha_{{\rm loc},a} = \min\left( \frac{10 h_a^2 A_a}{c^2_{{\rm s},a}}, \alpha_{\max} \right)$$

$$A_a = \xi_a \max\left[ -\frac{\rm d}{{\rm d}t} (\nabla \cdot \boldsymbol{v}_a), 0 \right]$$

$$\xi = \frac{|\nabla \cdot \boldsymbol{v}|^2}{|\nabla \cdot \boldsymbol{v}|^2 + |\nabla \times \boldsymbol{v}|^2}$$

# Conservation properties

Solver: exact conservation for

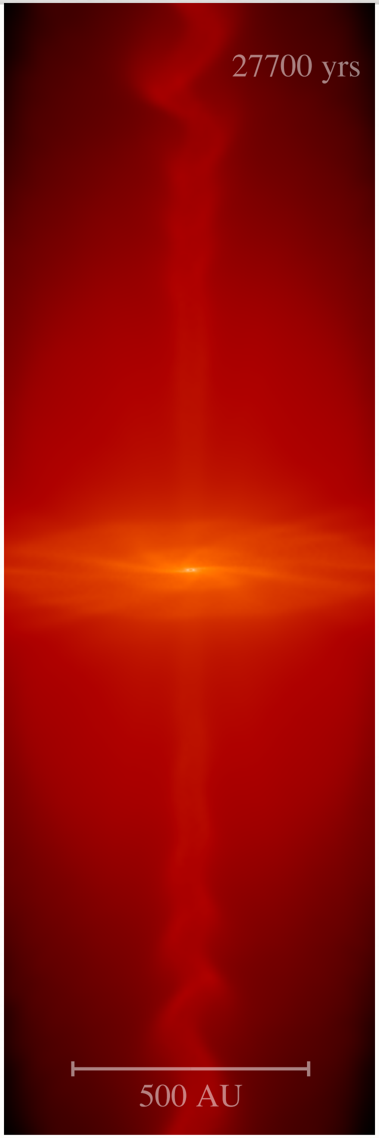
- linear momentum
- angular momentum
- energy

$$\frac{\mathrm{d}\boldsymbol{P}}{\mathrm{d}t} = \sum_a \sum_b m_a m_b \left[ \frac{P_a + q^a_{ab}}{\rho_a^2 \Omega_a} \nabla_a W_{ab}(h_a) + \frac{P_b + q^b_{ab}}{\rho_b^2 \Omega_b} \nabla_a W_{ab}(h_b) \right] = 0.$$

Symplectic time stepping (*Leap Frog*)

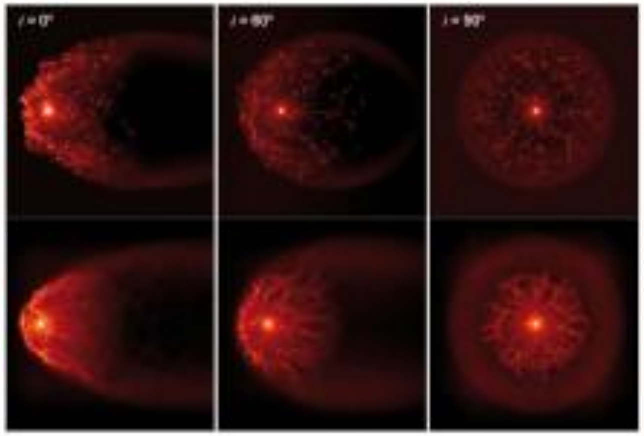
Individual timestepping:

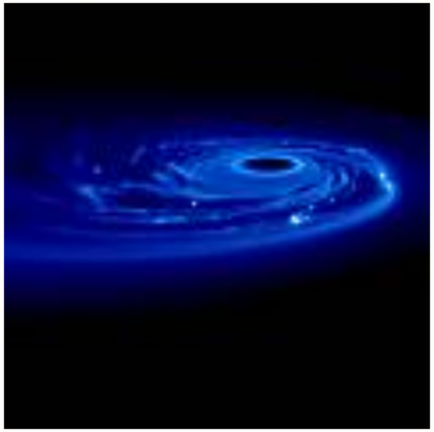
- breaks conservation
- keep stability

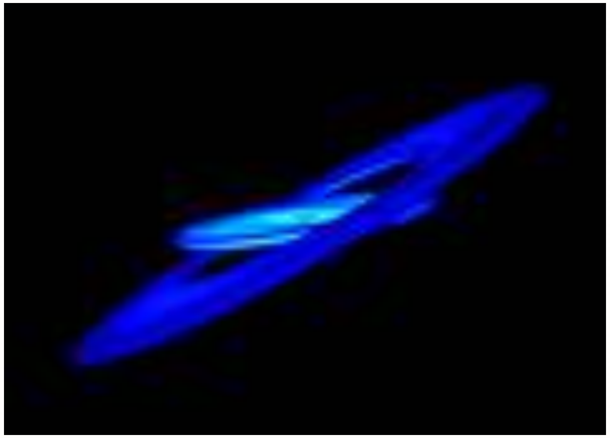


# Visualisation

Price (2007)







Designed for SPH, GIZA core

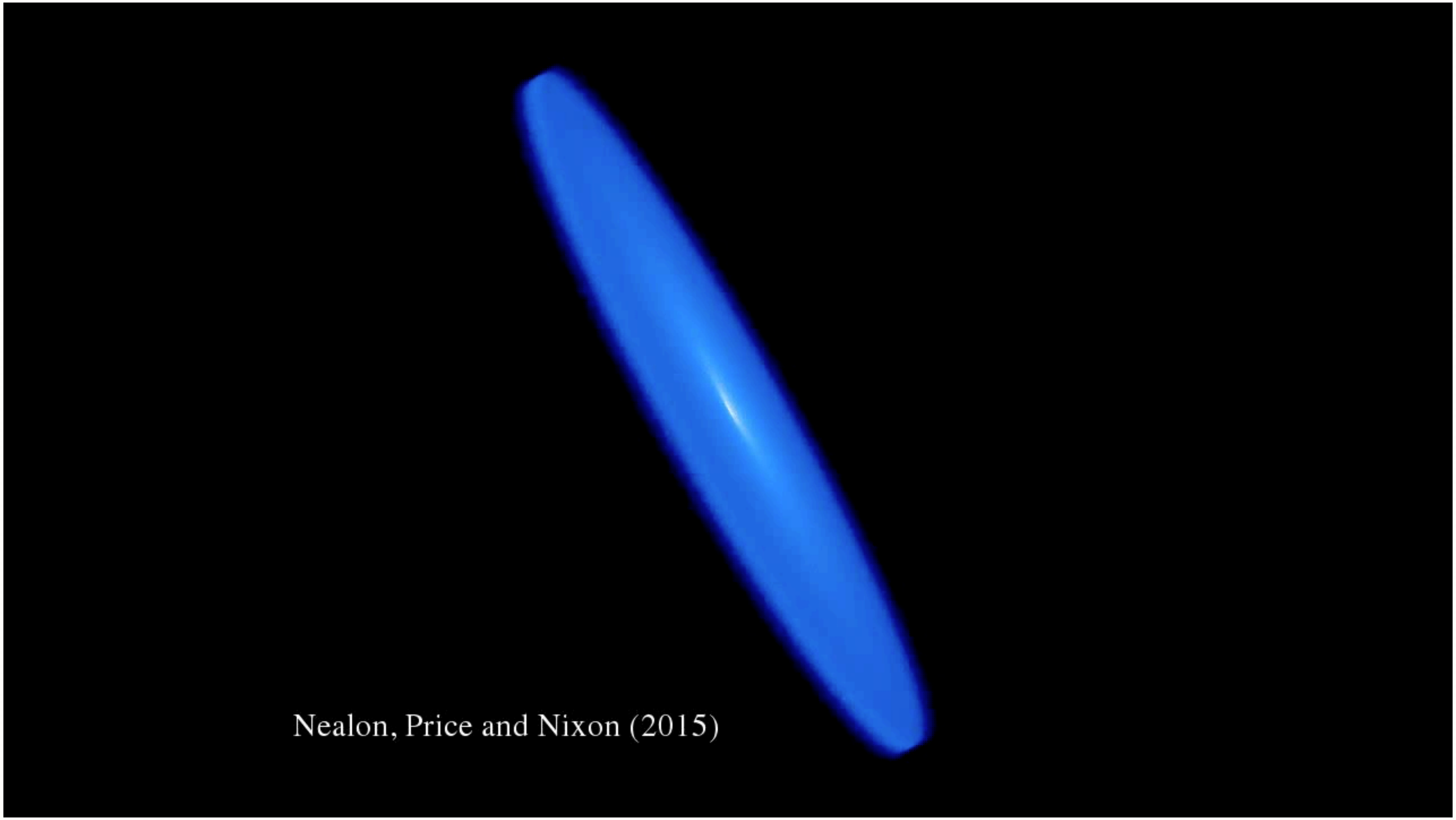
Public, GNU licence

Publication oriented *(plot format, plotting options, column density averaging, analytic solutions…)*

http://users.monash.edu.au/~dprice/splash/

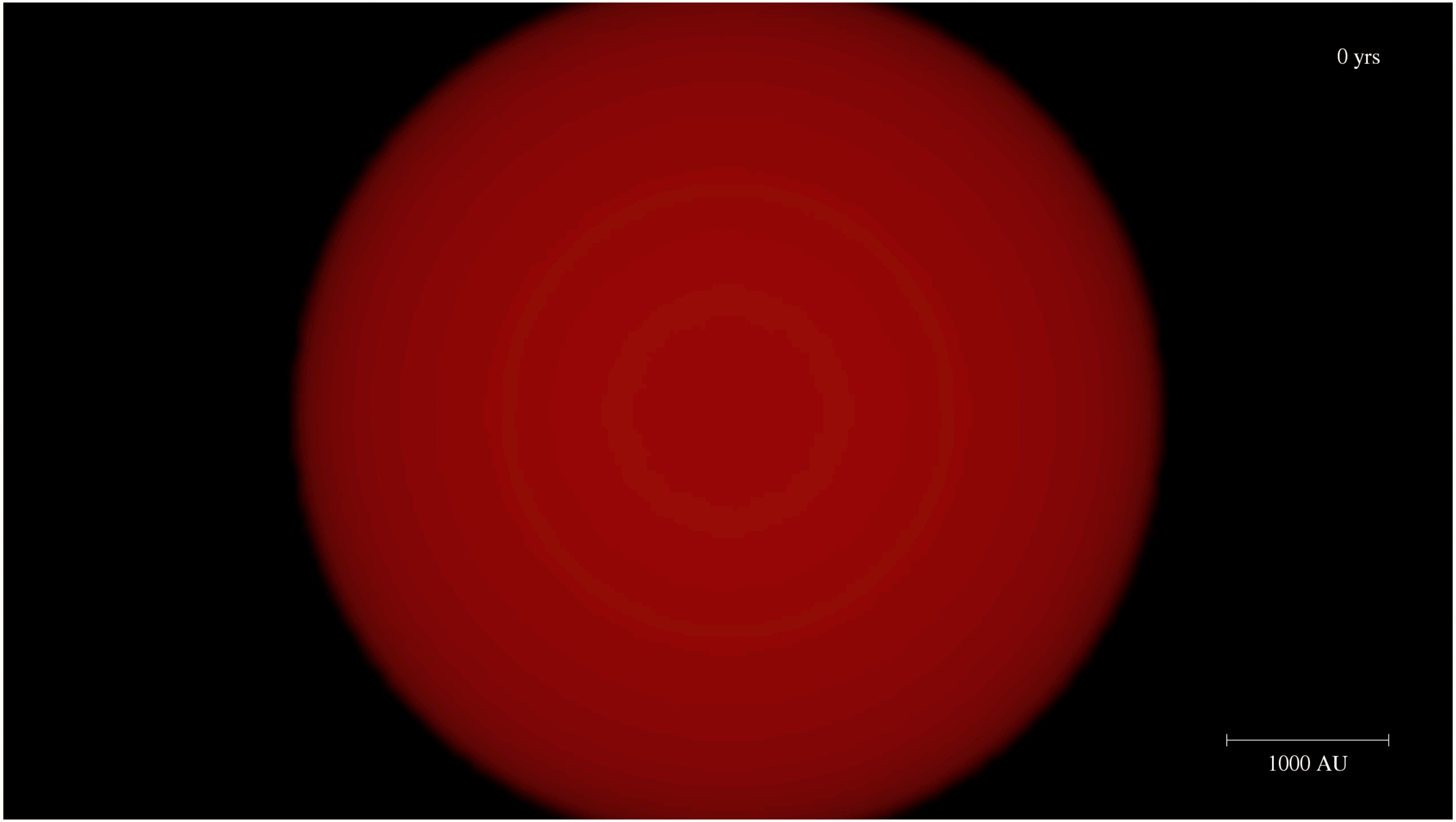
# Lense-Thirring precession

$$\boldsymbol{a}_{\mathrm{ext},a} = -\nabla\Phi_a + \boldsymbol{v}_a \times \boldsymbol{\Omega}_{p,a} \qquad \boldsymbol{\Omega}_{p,a} \equiv \frac{2\boldsymbol{S}}{|\boldsymbol{r}_a|^3} - \frac{6(\boldsymbol{S}\cdot\boldsymbol{r}_a)\boldsymbol{r}_a}{|\boldsymbol{r}_a|^5}$$




Nealon et al. (2015)

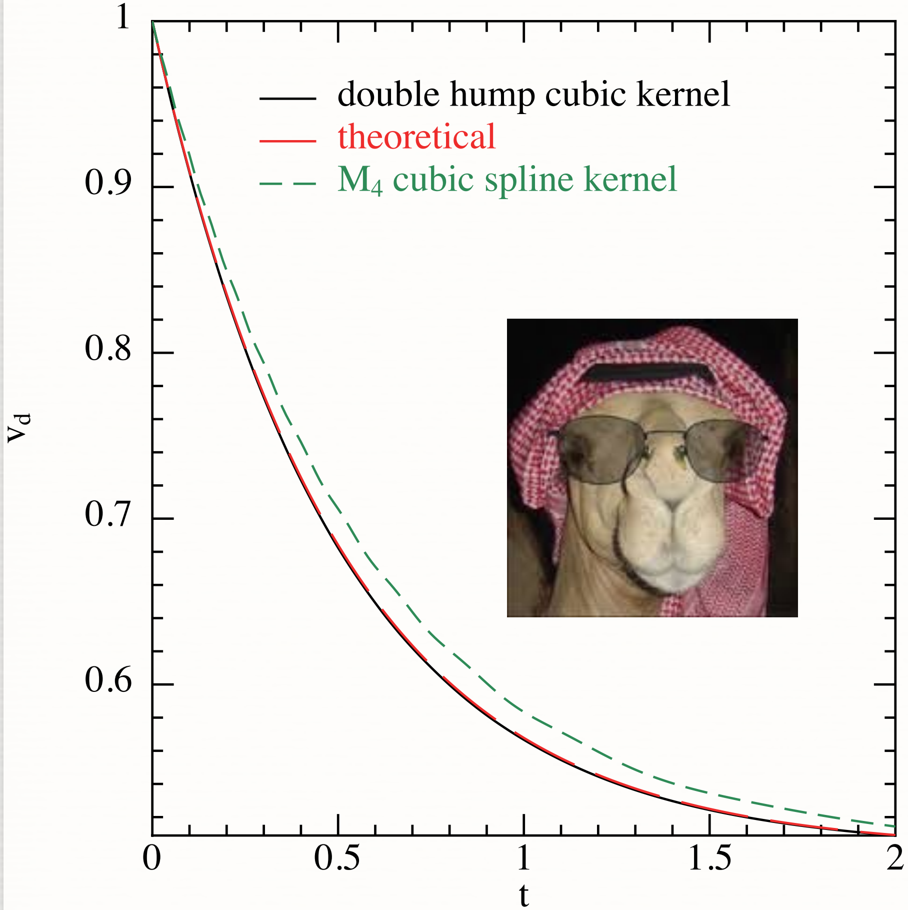
# Magnetic Jet



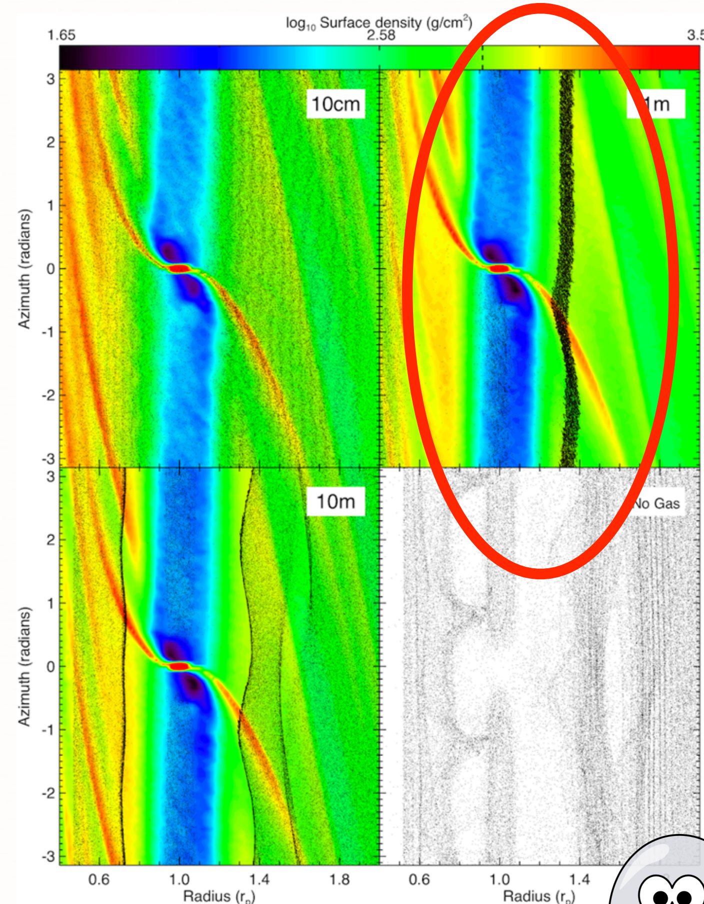

Price et al. (2012)

# Dust in detail: two-fluid algorithm

Laibe and Price (2011, 2012 a,b), Ayliffe et al. (2012)



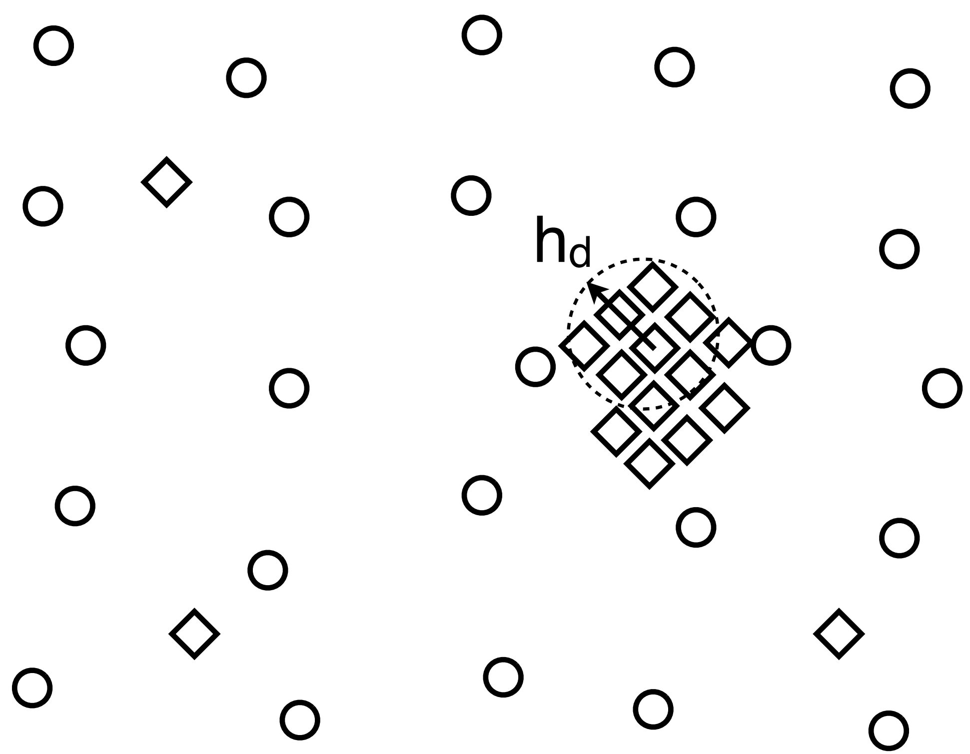

From dust in a 1D box…




…to a 3D disc

# Beware of artificial clumping !

Planet formation requires dust to concentrate a lot!

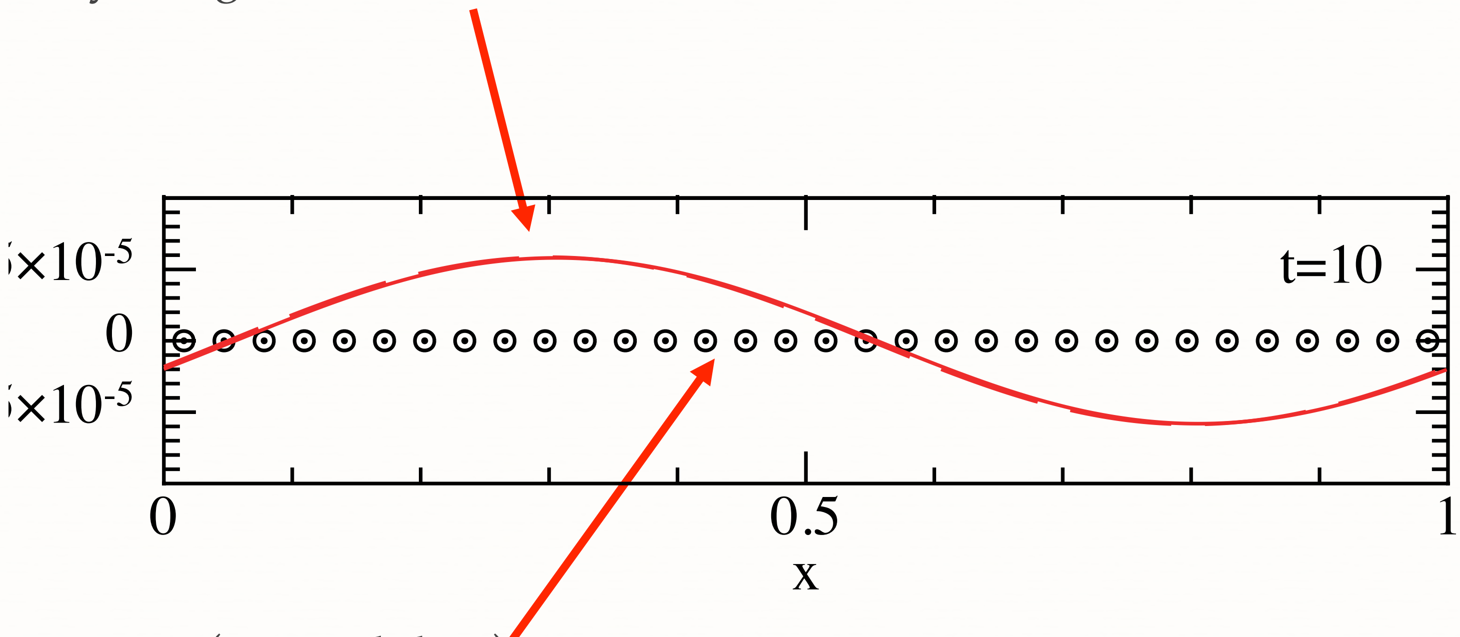



Dust below the gas resolution: artificial aggregates

# Simulating dust and gas

A sound wave in a mixture with small grains:

Analytics (gas and dust)




Numerics (gas and dust)

Numerics does not match analytics for small grains!

# Simulating dust and gas

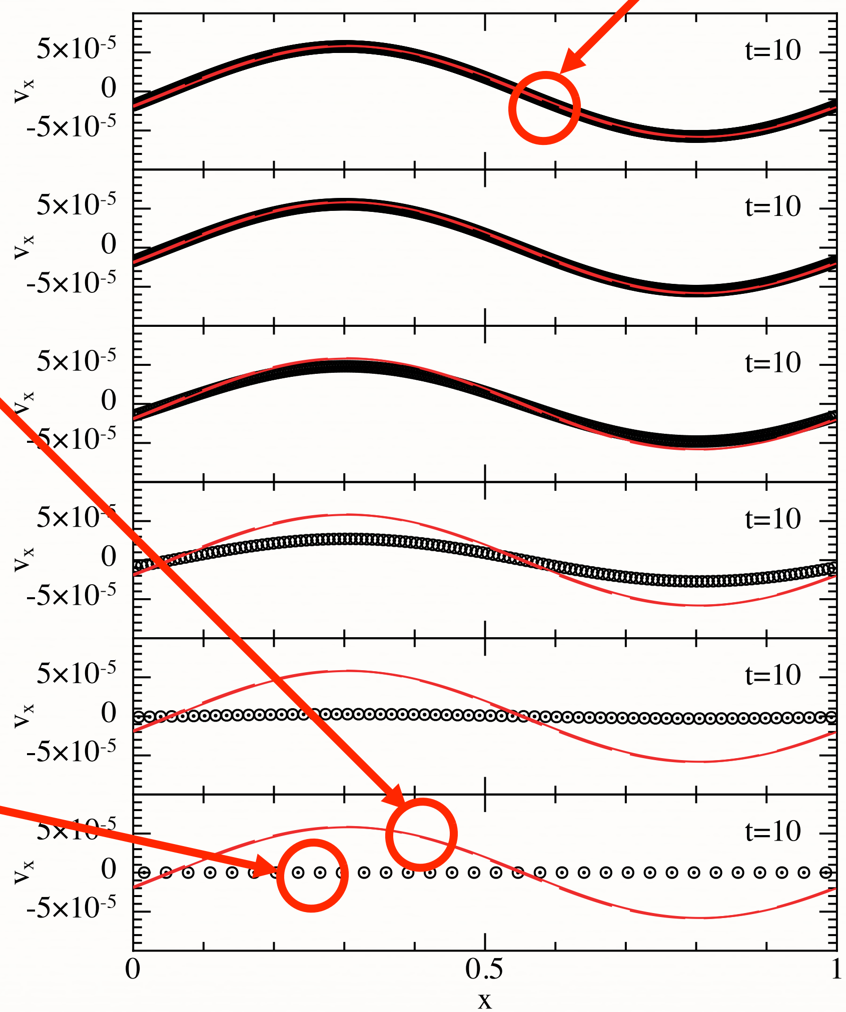
A sound wave in a mixture with small grains:

Very high-resolution: converged!

resolution (cost)

Gas and dust not quite superimposed

Energy hugely over-dissipated



# From a two fluid model for dust and gas mixtures...

Two fluids equations: mass, momentum and energy conservation:

Saffman (1962)

$$\frac{\partial \rho_{\rm g}}{\partial t} + \nabla.\left(\rho_{\rm g}\mathbf{v}_{\rm g}\right) = 0,$$

$$\frac{\partial \rho_{\rm d}}{\partial t} + \nabla.\left(\rho_{\rm d}\mathbf{v}_{\rm d}\right) = 0,$$

$$\rho_{\rm g}\left(\frac{\partial \mathbf{v}_{\rm g}}{\partial t} + \mathbf{v}_{\rm g}.\nabla\mathbf{v}_{\rm g}\right) = \rho_{\rm g}\mathbf{f} + K(\mathbf{v}_{\rm d} - \mathbf{v}_{\rm g}) - \nabla P_{\rm g},$$

$$\rho_{\rm d}\left(\frac{\partial \mathbf{v}_{\rm d}}{\partial t} + \mathbf{v}_{\rm d}.\nabla\mathbf{v}_{\rm d}\right) = \rho_{\rm d}\mathbf{f} - K(\mathbf{v}_{\rm d} - \mathbf{v}_{\rm g}),$$

$$\frac{\partial u}{\partial t} + (\mathbf{v}_{\rm g}\cdot\nabla)u = -\frac{P_{\rm g}}{\rho_g}(\nabla\cdot\mathbf{v}_{\rm g}) + K(\mathbf{v}_{\rm d} - \mathbf{v}_{\rm g})^2$$

We group the molecules/particles differently:

$$\rho \equiv \rho_{\rm g} + \rho_{\rm d} \qquad \mathbf{v} \equiv \frac{\rho_{\rm g}\mathbf{v}_{\rm g} + \rho_{\rm d}\mathbf{v}_{\rm d}}{\rho_{\rm g} + \rho_{\rm d}},$$

$$\epsilon = \rho_{\rm d}/\rho \qquad \Delta\mathbf{v} \equiv \mathbf{v}_{\rm d} - \mathbf{v}_{\rm g}$$

# ... to single fluid equations

Dual approach (no approximation):

$$\rho \equiv \rho_g + \rho_d \qquad \mathbf{v} \equiv \frac{\rho_g \mathbf{v}_g + \rho_d \mathbf{v}_d}{\rho_g + \rho_d}, \qquad \epsilon = \rho_d/\rho \qquad \Delta\mathbf{v} \equiv \mathbf{v}_d - \mathbf{v}_g$$

One fluid… …with two phases

$$\frac{d\rho}{dt} = -\rho(\nabla \cdot \mathbf{v}),$$

Total mass conserved

$$\frac{d\epsilon}{dt} = -\frac{1}{\rho}\nabla \cdot \left[\epsilon(1-\epsilon)\rho\Delta\mathbf{v}\right],$$

Composition evolution

$$\frac{d\mathbf{v}}{dt} = -\frac{\nabla P_g}{\rho} - \frac{1}{\rho}\nabla \cdot \left[\epsilon(1-\epsilon)\rho\Delta\mathbf{v}\Delta\mathbf{v}\right] + \mathbf{f},$$

Additional anisotropic pressure

$$\frac{d\Delta\mathbf{v}}{dt} = -\frac{\Delta\mathbf{v}}{t_s} + \frac{\nabla P_g}{(1-\epsilon)\rho} - (\Delta\mathbf{v}\cdot\nabla)\mathbf{v} + \frac{1}{2}\nabla\left[(2\epsilon-1)\Delta\mathbf{v}^2\right],$$

Trivial dissipation term

$$\frac{du}{dt} = -\frac{P_g}{(1-\epsilon)\rho}\nabla\cdot(\mathbf{v}-\epsilon\Delta\mathbf{v}) + \epsilon(\Delta\mathbf{v}\cdot\nabla)u + \epsilon\frac{\Delta\mathbf{v}^2}{t_s},$$

Energy conserved

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{v}.\nabla$$

Laibe and Price (2014 a,b)

# Single fluid, diffusion limit

Laibe and Price (2014 a,b)
Price and Laibe (2015)

Two fluids:

Dust Gas

→

One fluid

Diffusion limit:

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\rho(\nabla \cdot \boldsymbol{v}),$$

$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = -\frac{\nabla P}{\rho} + \boldsymbol{f},$$

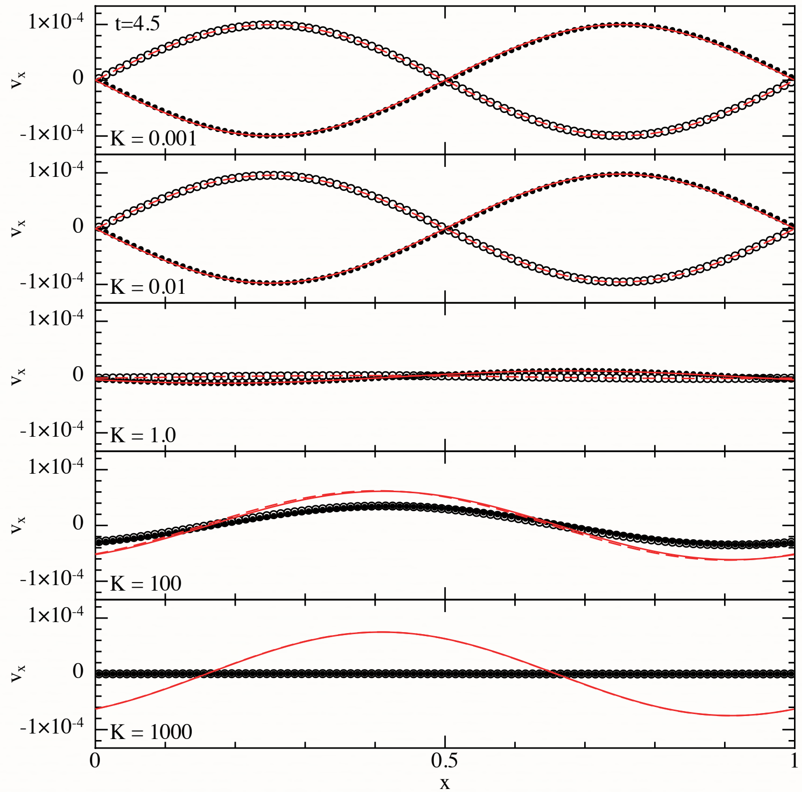
$$\frac{\mathrm{d}\epsilon}{\mathrm{d}t} = -\frac{1}{\rho}\nabla \cdot (\epsilon t_{\mathrm{s}} \nabla P)\,,$$

$$\frac{\mathrm{d}u}{\mathrm{d}t} = -\frac{P}{\rho_{\mathrm{g}}}(\nabla \cdot \boldsymbol{v}) - \frac{\epsilon t_{\mathrm{s}}}{\rho_{\mathrm{g}}}(\nabla P \cdot \nabla u) + \Lambda_{\mathrm{heat}} - \Lambda_{\mathrm{cool}}.$$

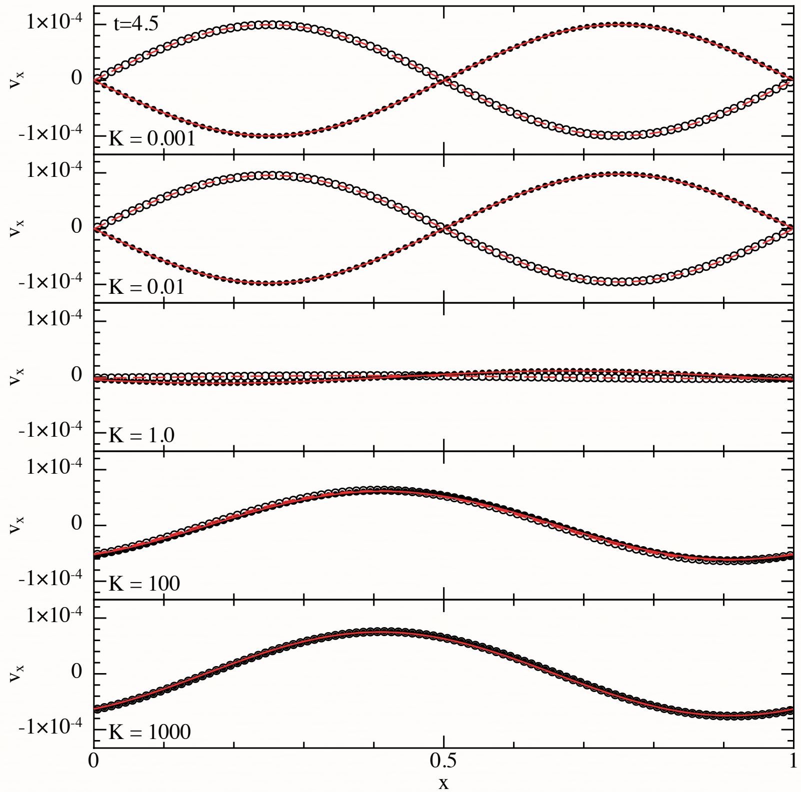
→

# Dustywave

## two fluids formalism

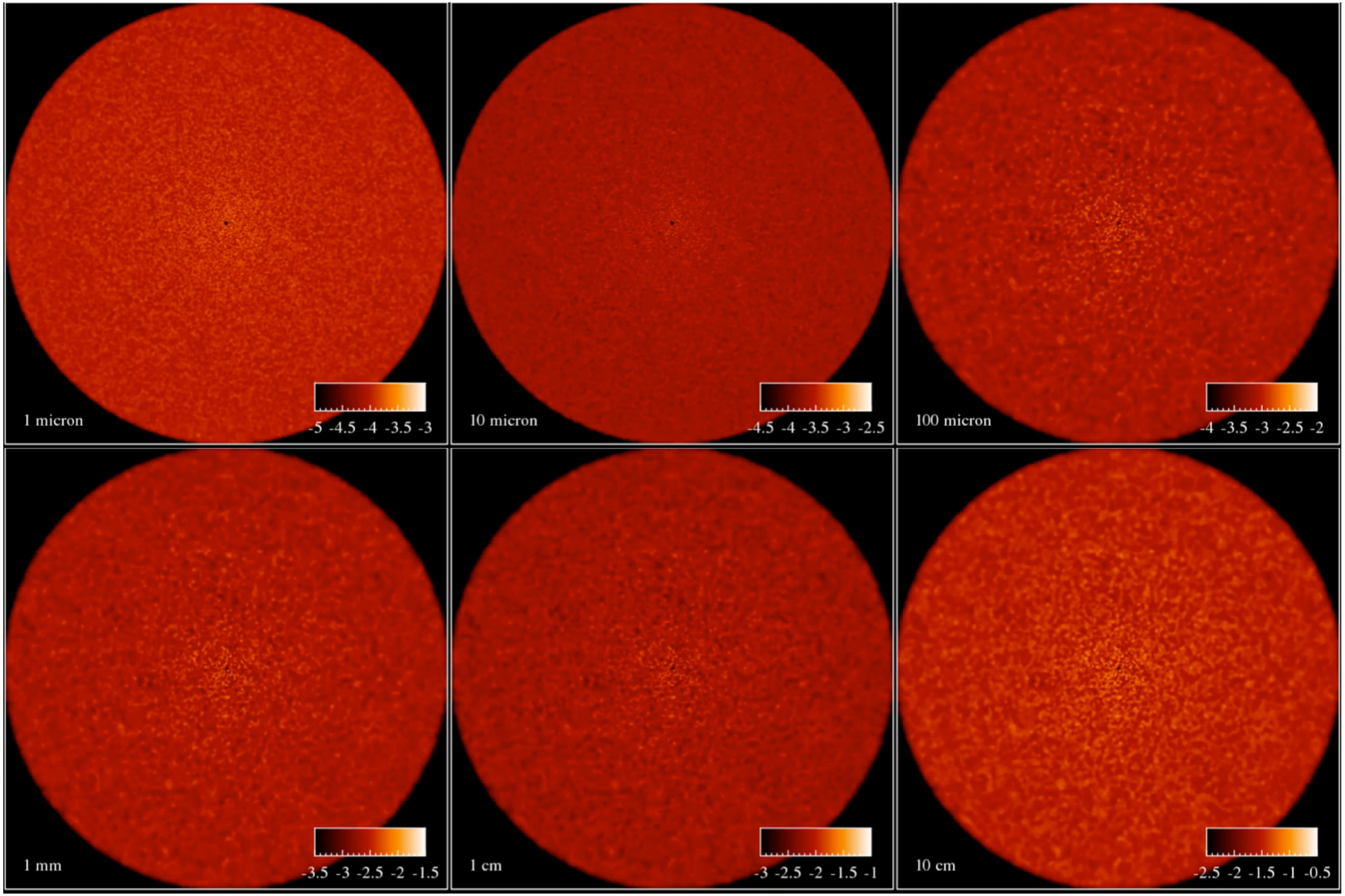



## one fluid formalism



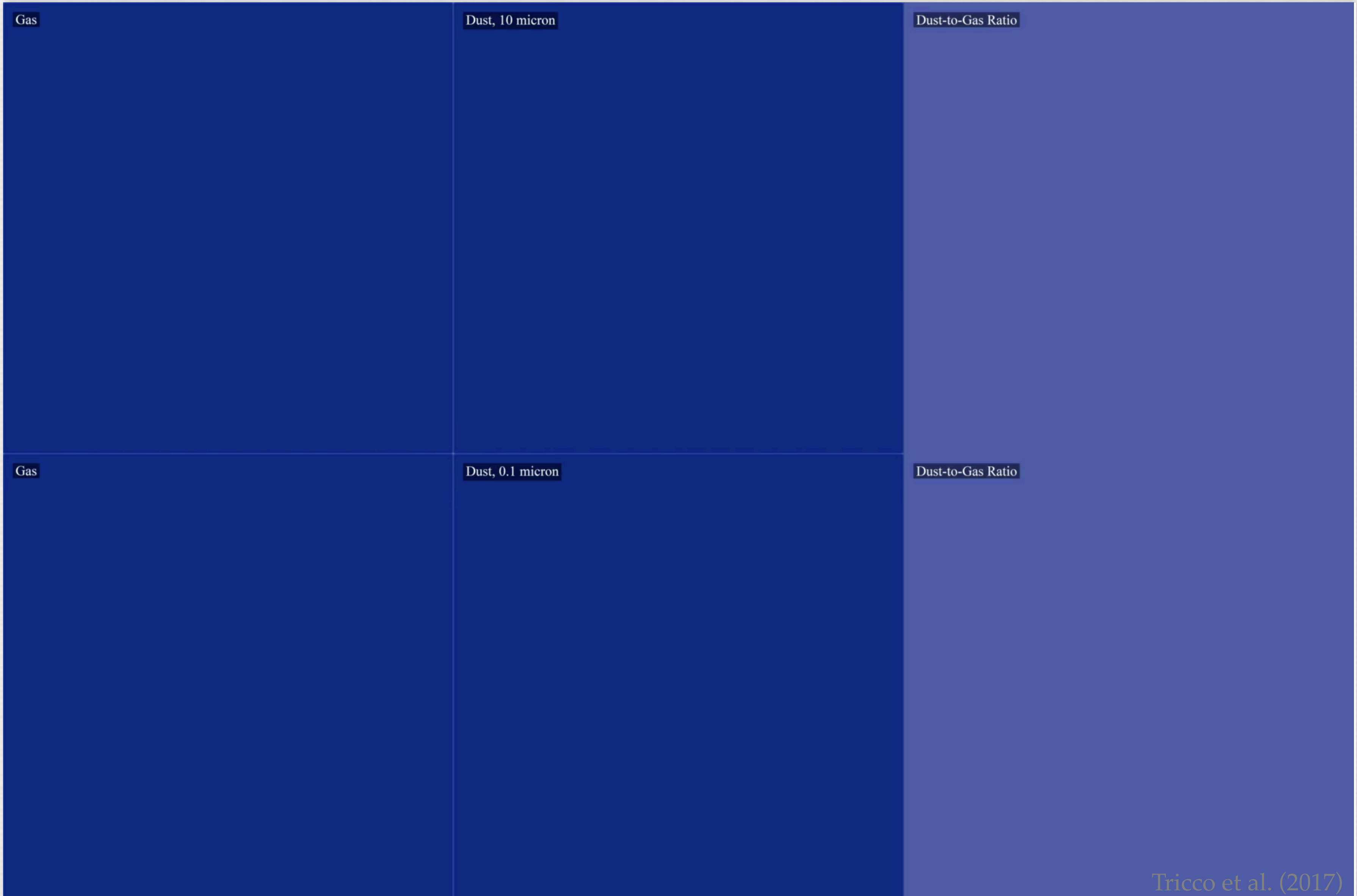

Careful: very large grains…

# HL Tau



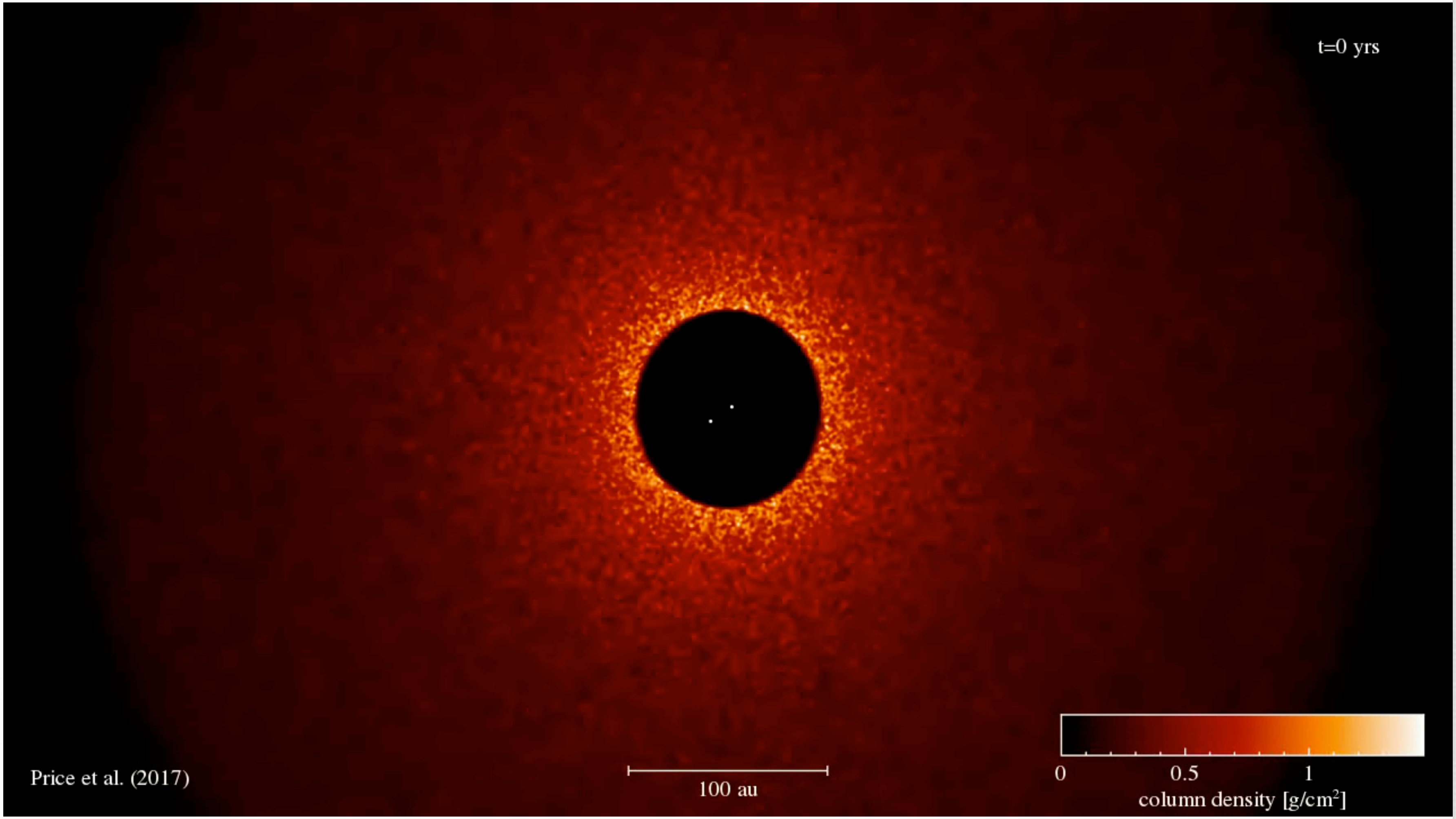

Dipierro et al. (2015)

# Turbulence in the ISM




Tricco et al. (2017)

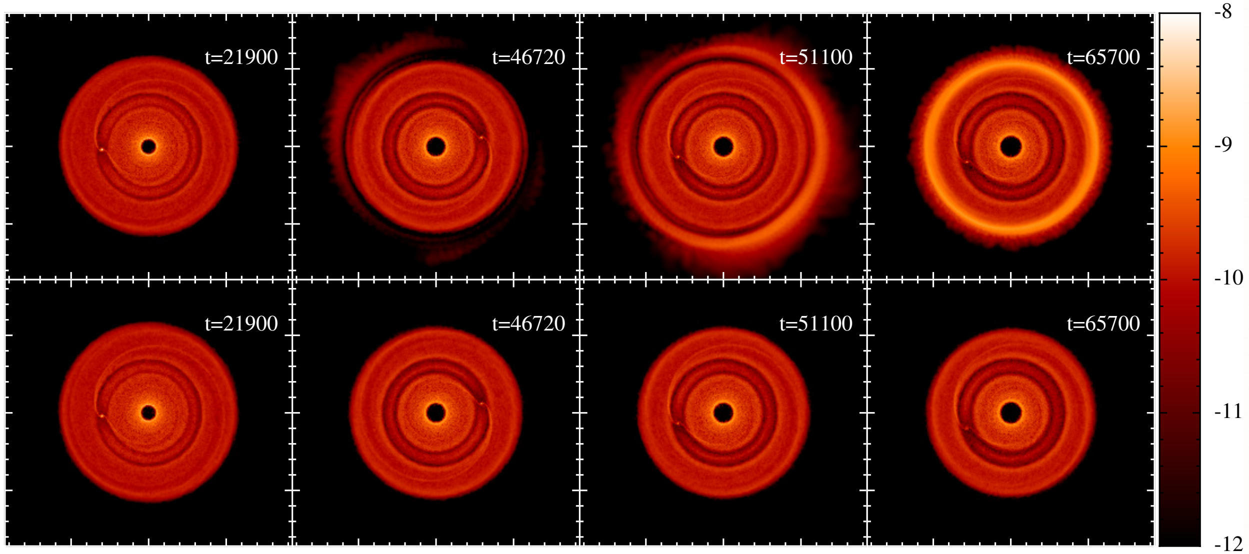
# HD 142527




Price et al. (2018)

# Updates on the algorithms

Regularisation:
(public)

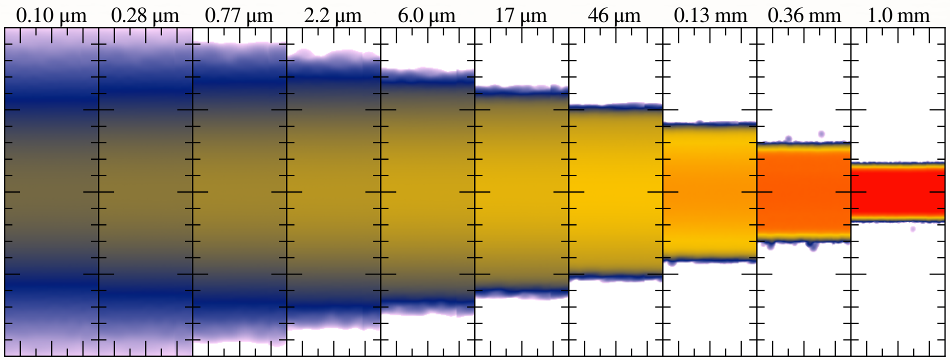



Ballabio al. (2018)

MULTIGRAIN
(private)

=

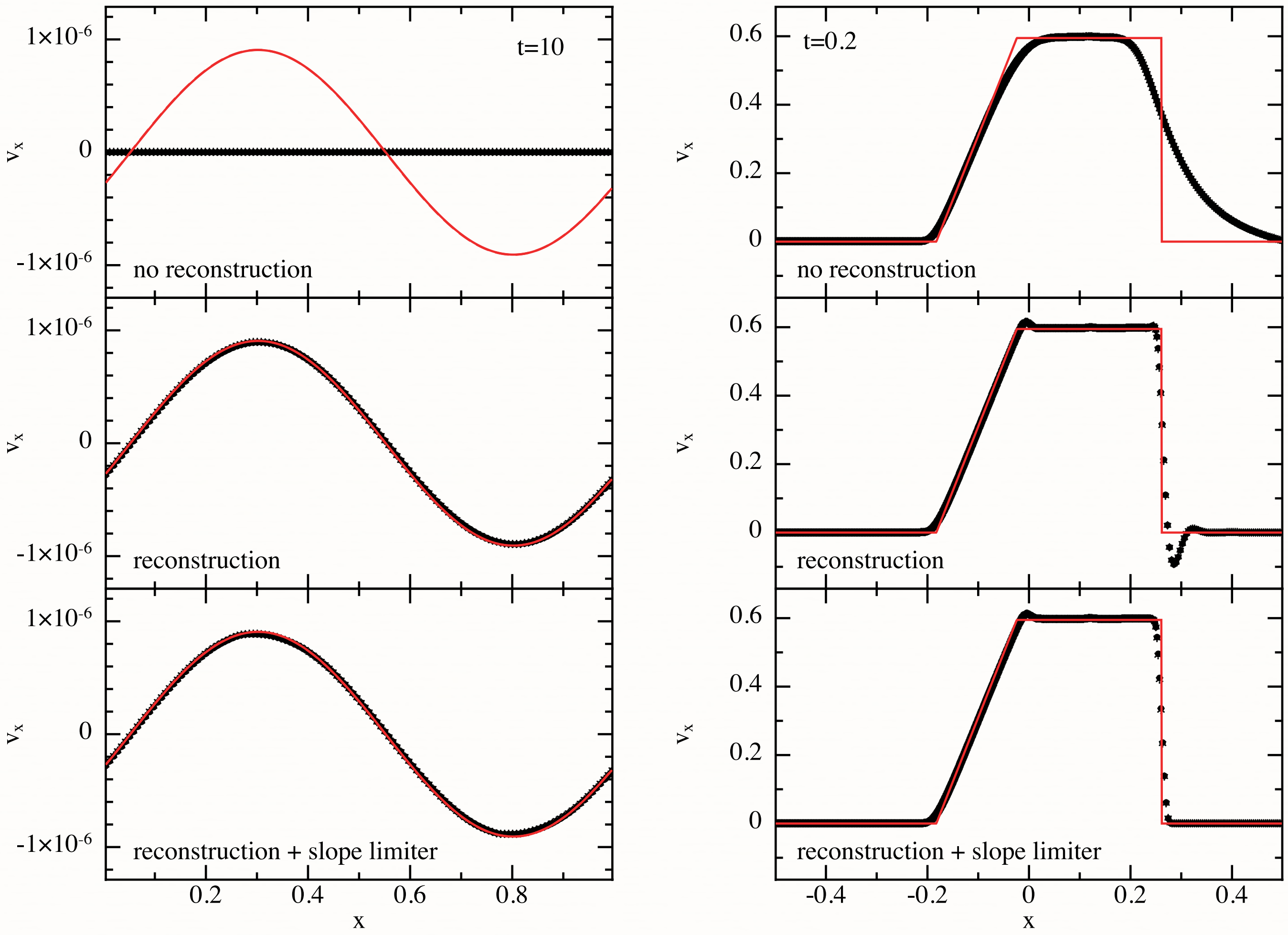
**0.1 μm** → **1.0 mm**




Hutchinson et al. (2018)

# Back to two fluids

Price and Laibe (in prep.)




*DUSTYWAVE*

*DUSTYSHOCK*

# In practice

The *PHANTOM* paper (Price et al. 2018):

Phantom: A smoothed particle hydrodynamics and magnetohydrodynamics code for astrophysics

The *PHANTOM* bitbucket:

https://phantomsph.bitbucket.io/#home

The *PHANTOM* Wiki:

https://bitbucket.org/danielprice/phantom/wiki/Home

The *PHANTOM* Slack:

https://phantomsph.slack.com

Teaser: PHANTOM+MCFOST = live radiative transfer