Simulations of magnetic stars

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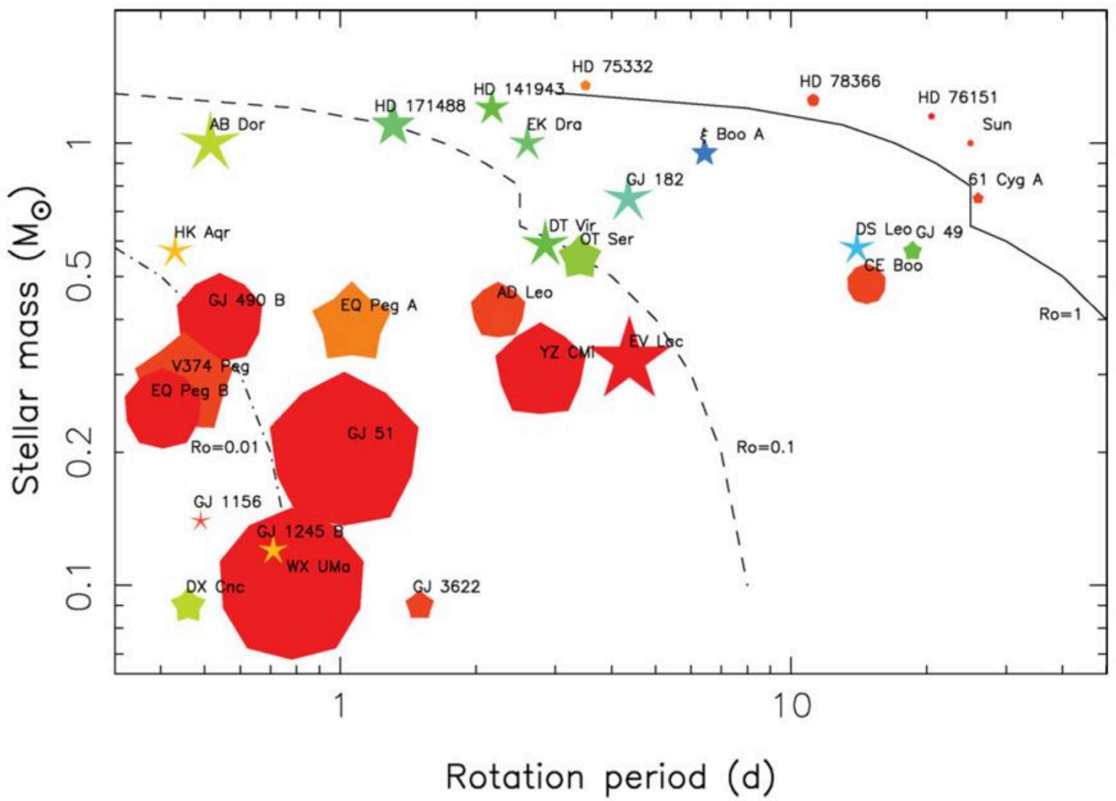
Lyon , October 11, 2018

Magnetic fields in cool stars

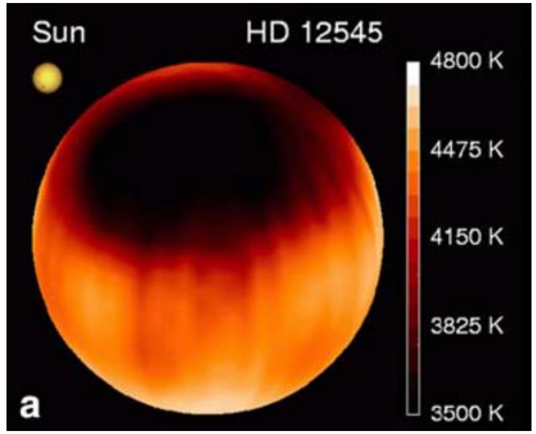
Morin, Donati et al. (2008-2010), Folsom et al. 2016



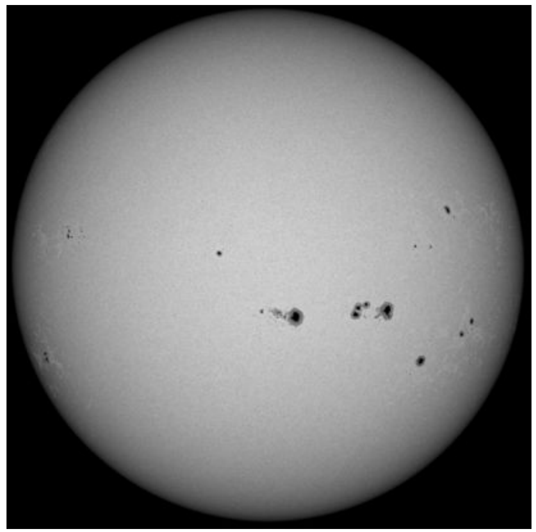
- ❑ Mostly multipolar for $M_\odot > 0.35$
- ❑ Mostly dipolar for $M_\odot < 0.35$
- ❑ Field strength increases with rotation
- ❑ More and more toroidal with rotation

Petit et al. 2008, B cool survey (Marsden et al. 2014)

Strassmeier (1999)

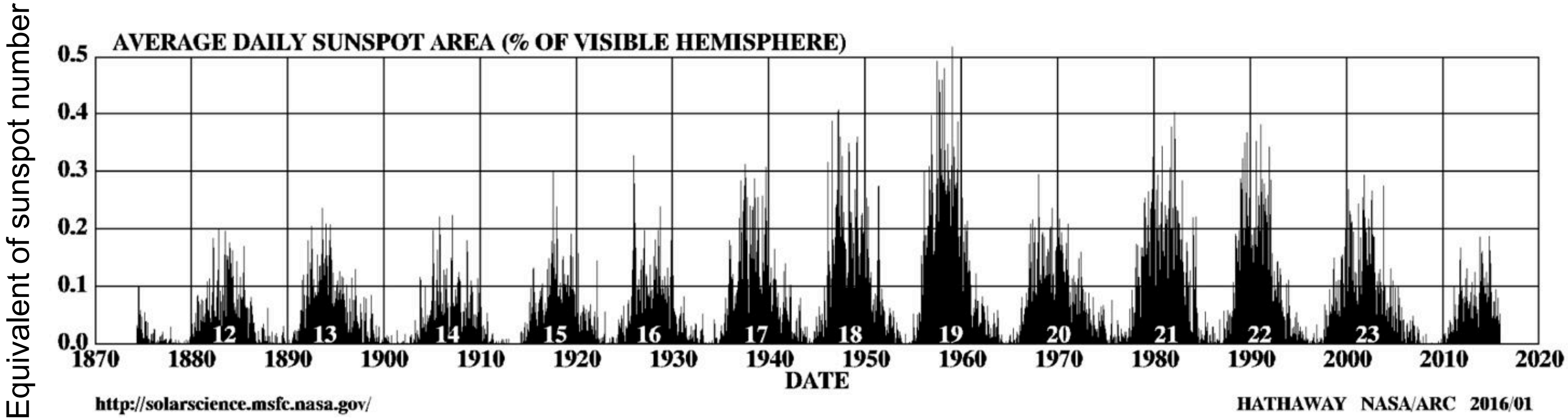


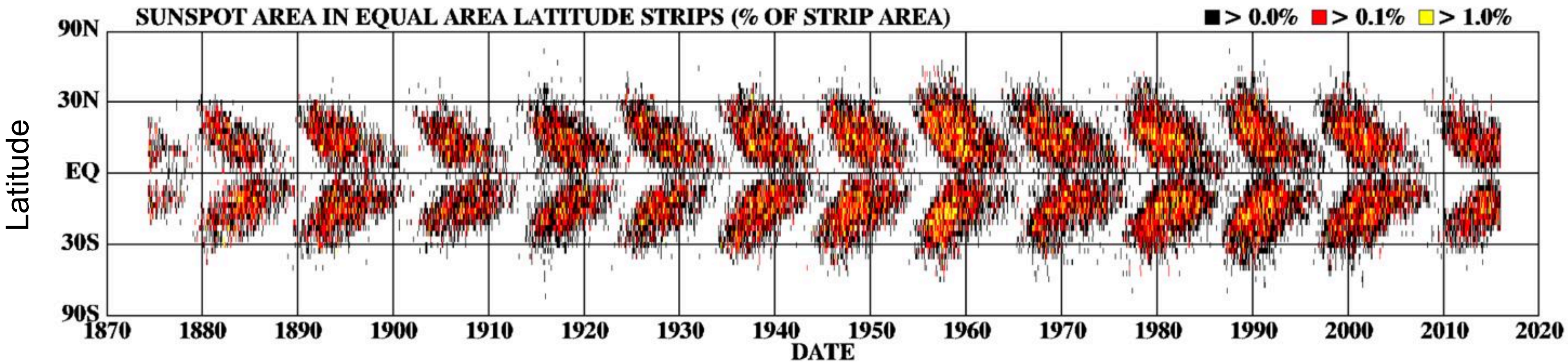
SDO data (July 2014)



- ❑ In stars cooler than the Sun: Polar spots with large coverage

Sunspots: temporal evolution

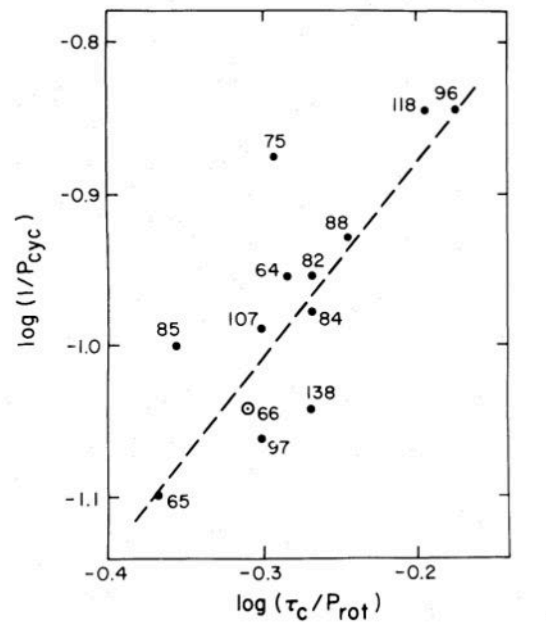




Observations of magnetic cycles on other stars

- **Indirect measurements**: chromospheric activity

Noyes et al. 1984



Chromospheric activity (Mount Wilson data, Ca II HK lines): $P_{cyc}=Ro^{1.28+/-0.48}$

where the Rossby number $Ro=P_{rot}/\tau$

=> P_{cyc} increases with P_{rot}

- **Recent direct measurements**: magnetic field

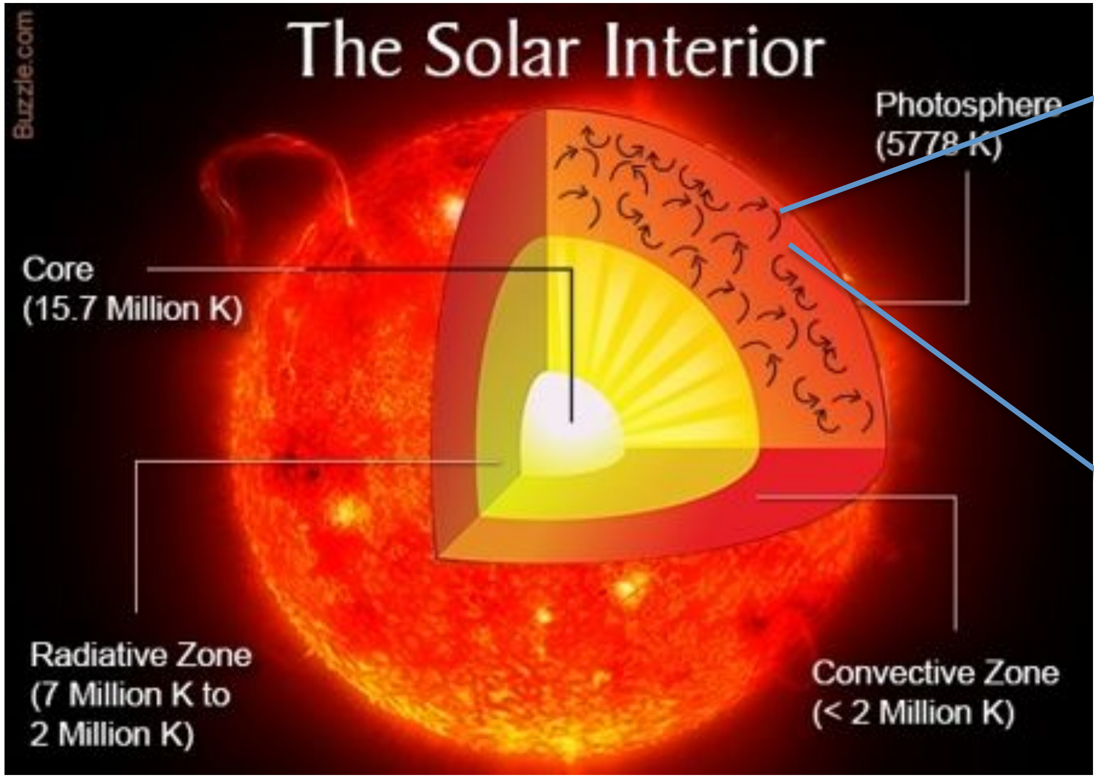
Donati et al 2008, Fares et al 2009, Mengel et al 2016: τ boo: 2 years

Petit et al 2009, Morgenthaler et al 2011: HD 190771 (complex variability)

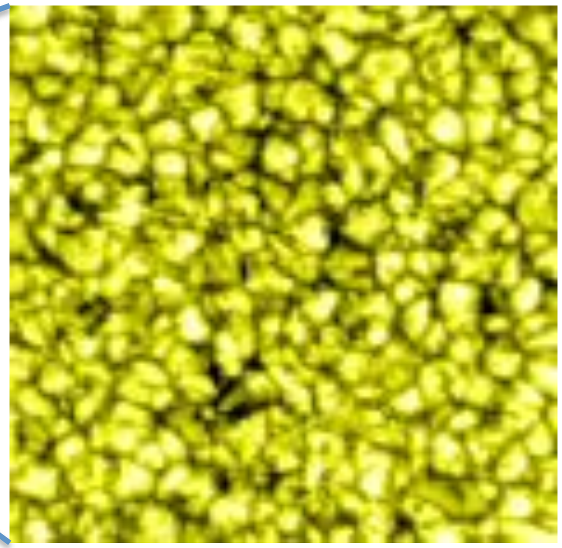
Garcia et al 2010, Salabert et al. 2016, Kiefer et al. 2017: asteroseismic signatures

Boro-Saika et al 2016: 61 Cyg A (solar twin): 14 years

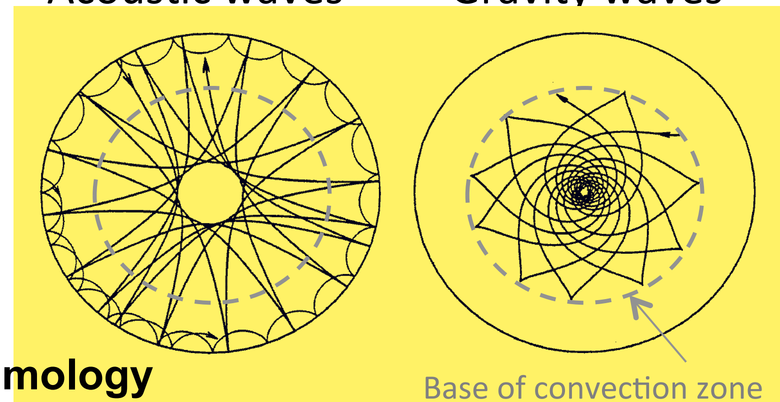
Solar interior and plasma flows



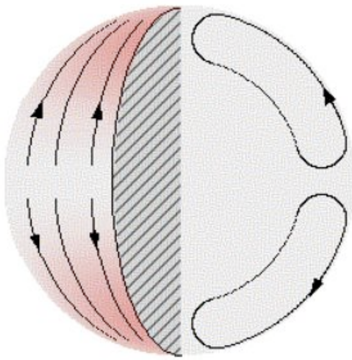
- ❑ Granulation (surface convection)



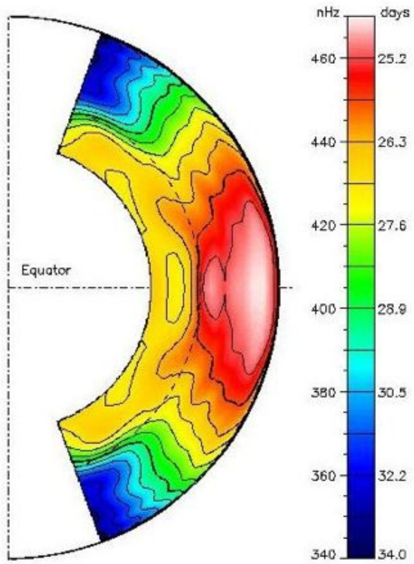
Acoustic waves Gravity waves



Helioseismology



- ❑ Meridional flow



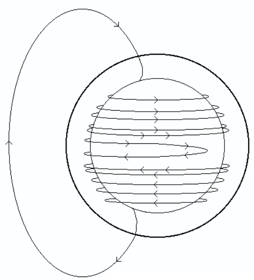
- ❑ Rotation

Theory: the induction equation (MHD)

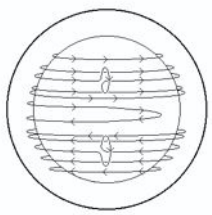
$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) - \nabla \times (\eta \nabla \times \mathbf{B})$$

$$\frac{\partial \mathbf{B}}{\partial t} = \underbrace{(\mathbf{B} \cdot \nabla)\mathbf{u}}_{Shearing\, of\, B} - \underbrace{(\mathbf{u} \cdot \nabla)\mathbf{B}}_{Advection\, of\, B} - \underbrace{\mathbf{B}(\nabla \cdot \mathbf{u})}_{Compressibility} - \underbrace{\nabla \times (\eta_m \nabla \times \mathbf{B})}_{Magnetic\, diffusion}$$

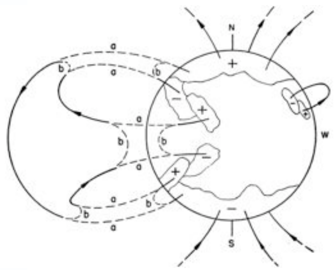
Source of magnetic field



✓Ω-effect

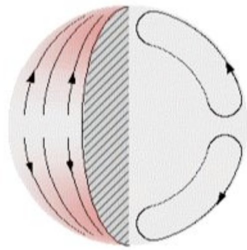


✓α-effect



✓ Babcock-Leighton source term

Transport of magnetic field



- ✓ **Large-scale flows (meridional circulation)**
- ✓ **Downward pumping by penetrative convection**
- ✓ **Transport from the base of the convection zone to the surface**

2D numerical simulations

Mean induction equation

- Simplified description of physical processes
- Fast and efficient tool
  Parametric studies

3D numerical simulations

Full MHD equations

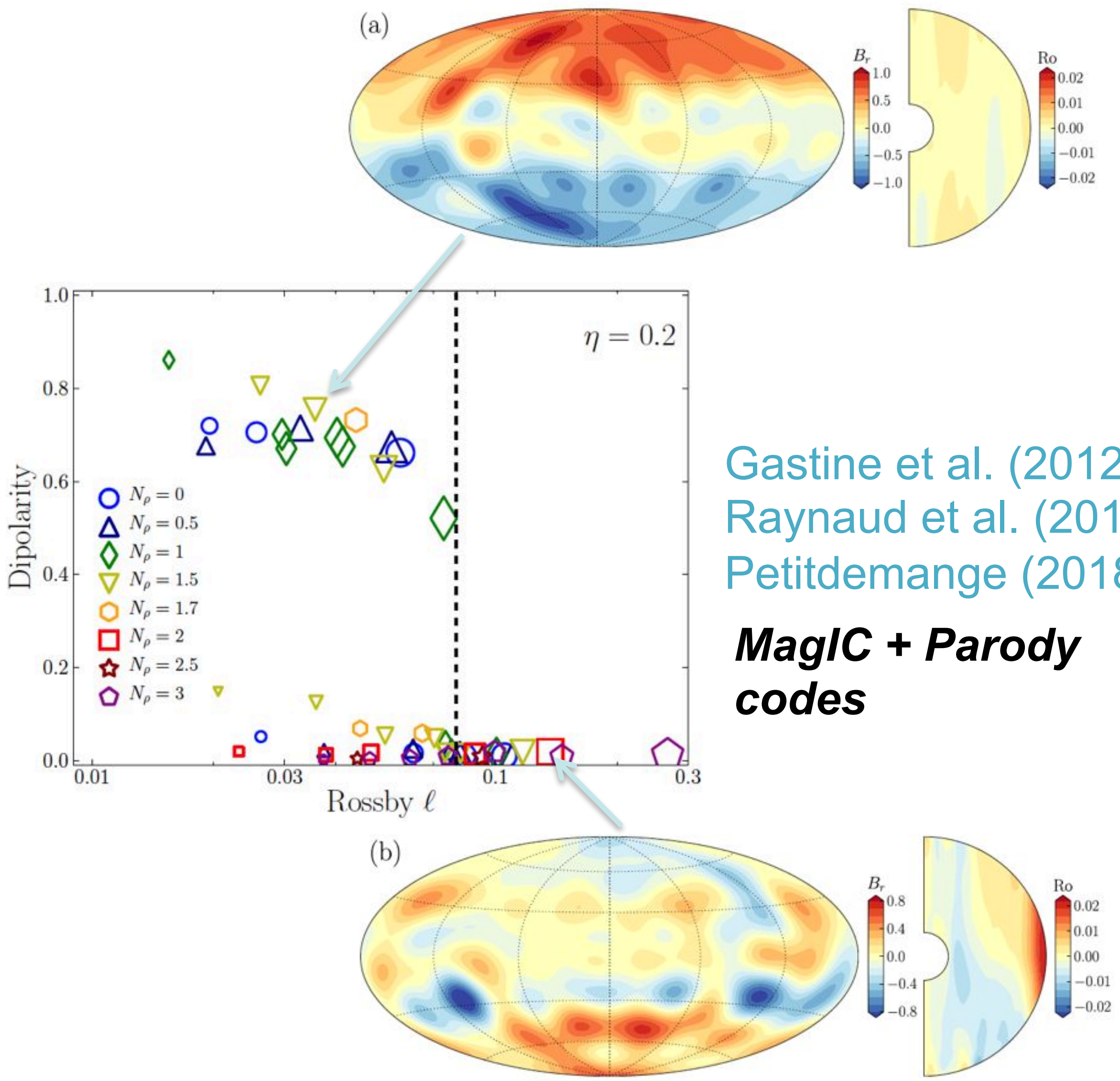
- Much more complex
- Self-consistent simulations

Magnetic topology: influence of the Rossby number

- ❑ Change in Rossby Ro=inertia/Coriolis (also seen in planetary dynamos: Christensen & Aubert 2006)

  - Small Ro: Ordering role of Coriolis=dipolar (no role of shear)

  - Large Ro: Inertia becomes dominant=multipolar (important role of shear)

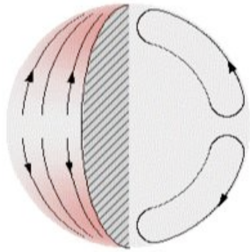


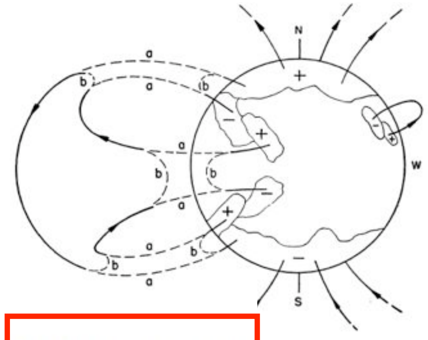
Gastine et al. (2012, 201
Raynaud et al. (2015)
Petitdemange (2018)

MagIC + Parody codes

Magnetic cycles in 2D models

- Mean-field induction equation only
- Babcock-Leighton dynamo model
- 2 coupled PDEs

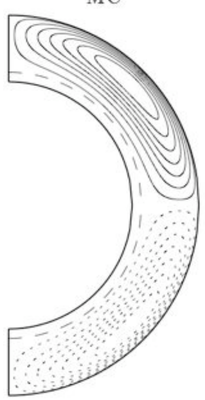




$$\frac{\partial A_\phi}{\partial t} = \frac{\eta}{\eta_t}\left(\nabla^2 - \frac{1}{\varpi^2}\right)A_\phi - R_e \frac{\mathbf{u}_p}{\varpi}\cdot\nabla(\varpi A_\phi) + \cancel{C_\alpha \alpha B_\phi} + \boxed{C_s S(r,\theta,B_\phi)}$$

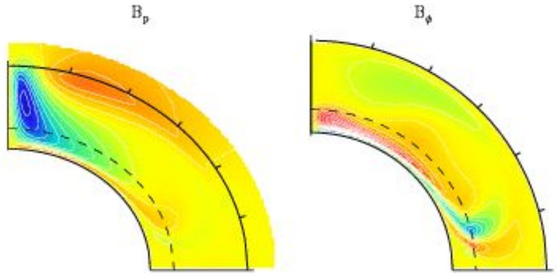
$$\frac{\partial B_\phi}{\partial t} = \frac{\eta}{\eta_t}\left(\nabla^2 - \frac{1}{\varpi^2}\right)B_\phi + \frac{1}{\varpi}\frac{\partial(\varpi B_\phi)}{\partial r}\frac{\partial(\eta/\eta_t)}{\partial r} - R_e \varpi \mathbf{u}_p \cdot \nabla\left(\frac{B_\phi}{\varpi}\right) - R_e B_\phi \nabla\cdot\mathbf{u}_p + C_\Omega \varpi(\nabla\times(\varpi A_\phi \hat{\mathbf{e}}_\phi))\cdot\nabla\Omega$$

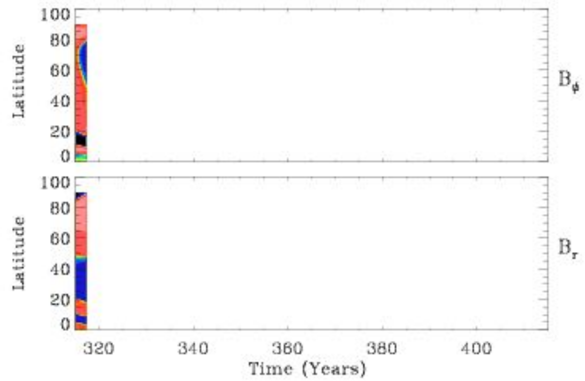
Standard model:
single-celled
meridional
circulation



Dikpati & Charbonneau 1999
Jouve & Brun 2007

STELEM code





- Cyclic field
- Butterfly diagram

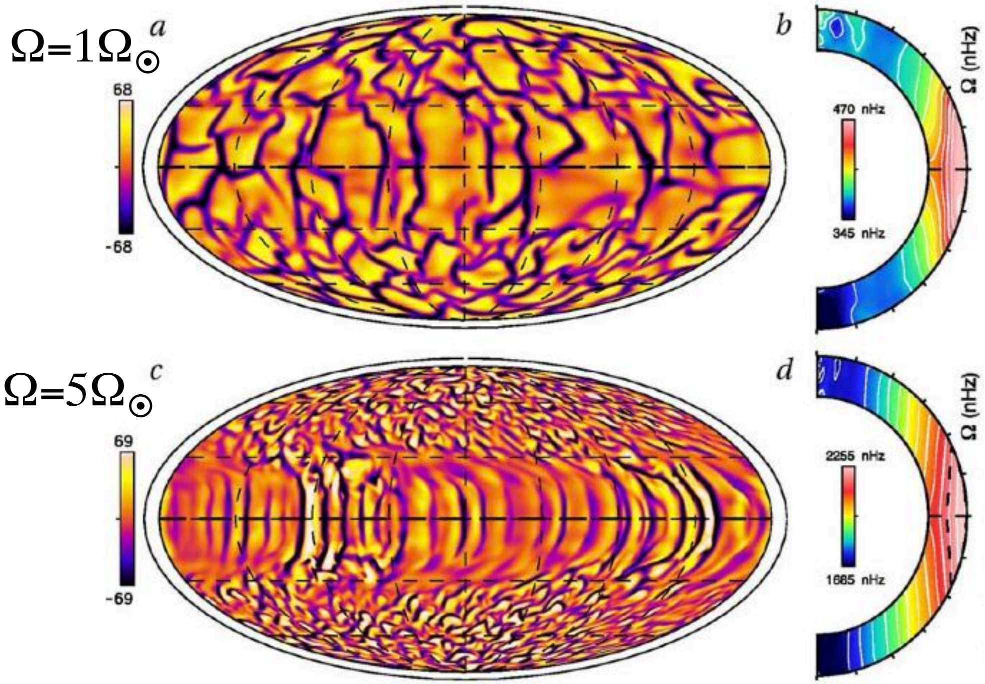
ok with observations

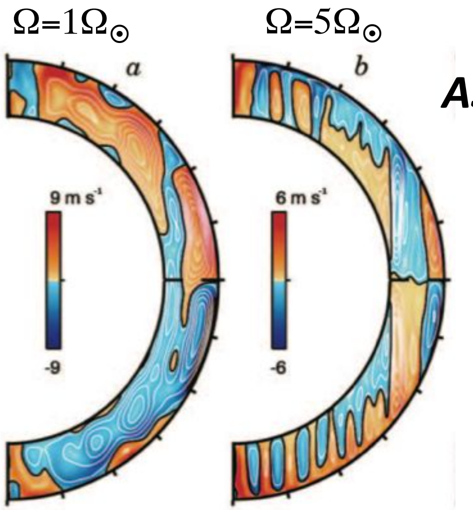
- Very strong dependence of cycle period on MC amplitude

$$P_{cyc} = \boxed{v_0^{-0.91}} s_0^{-0.013} \eta^{-0.075} \Omega_0^{-0.014}$$

Is this solar model applicable for rapidly-rotating solar-like stars?

Prescriptions from 3D models

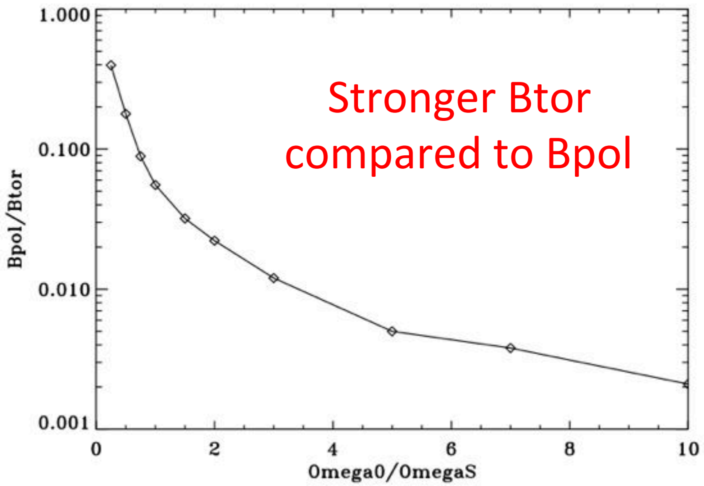


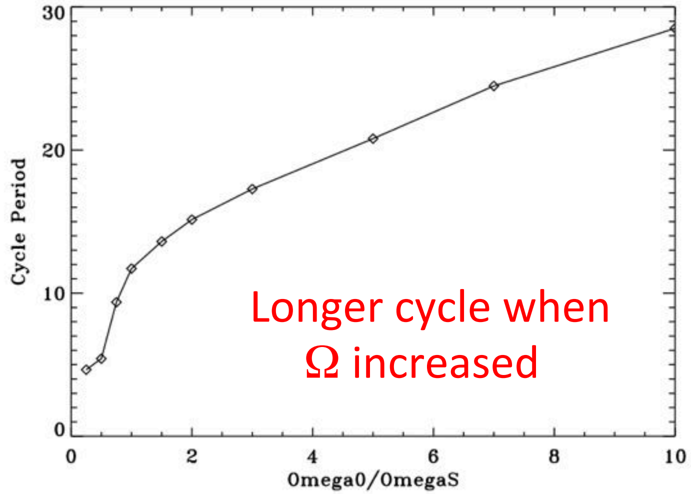


ASH code

Prescriptions from Brown et al. 2008:

- Vp α $\Omega^{-0.9}$
- $\Delta\Omega$ increases with Ω





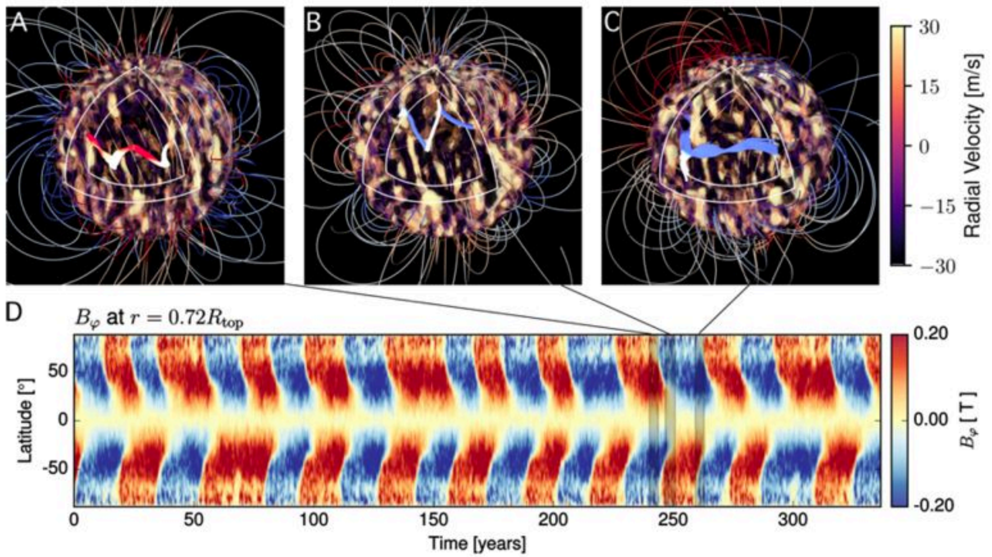
Jouve et al. 2010

The MC profile needs to be modified to reconcile models and observations

Applying solar models to other stars: more realistic models

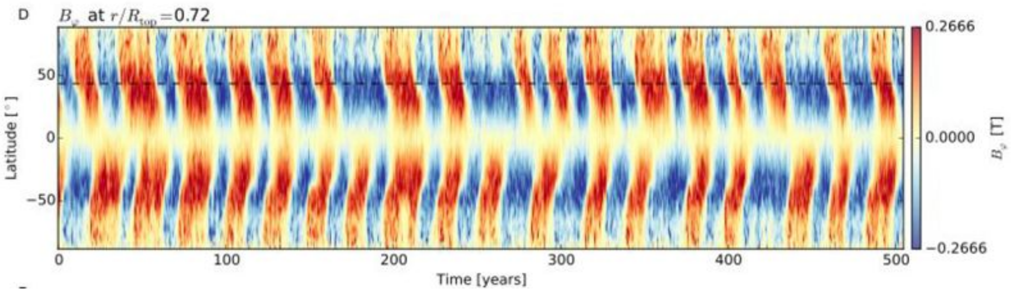
Eulag code

$\Omega = \Omega_\odot$

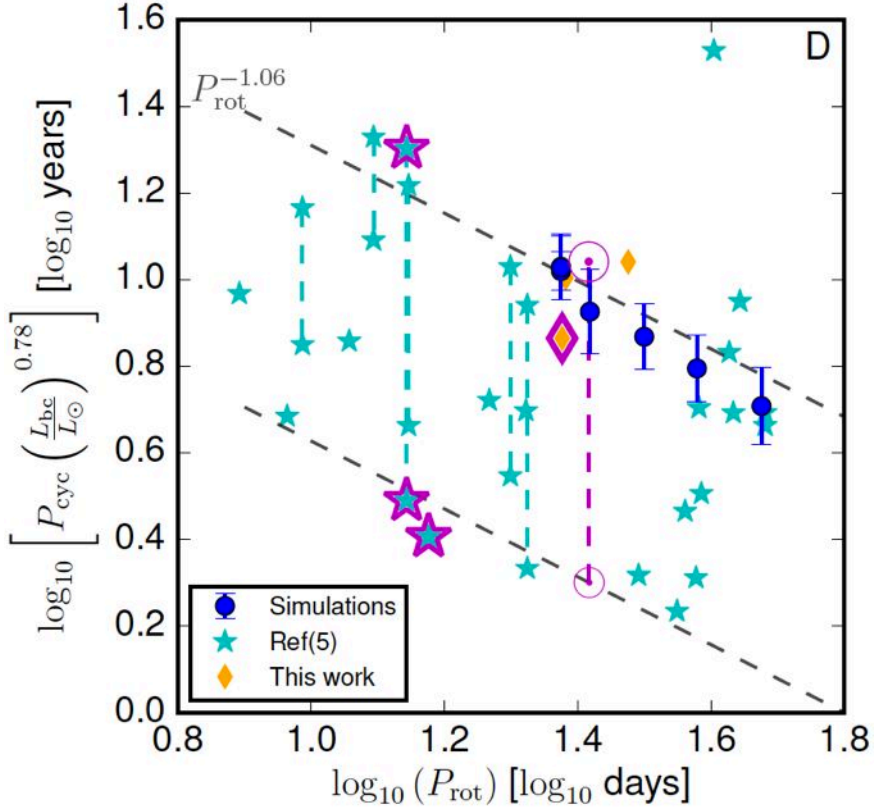


At fixed luminosity,
slower rotation produces
shorter magnetic cycles!

$\Omega = 0.6\Omega_\odot$



Strugarek et al. 2017

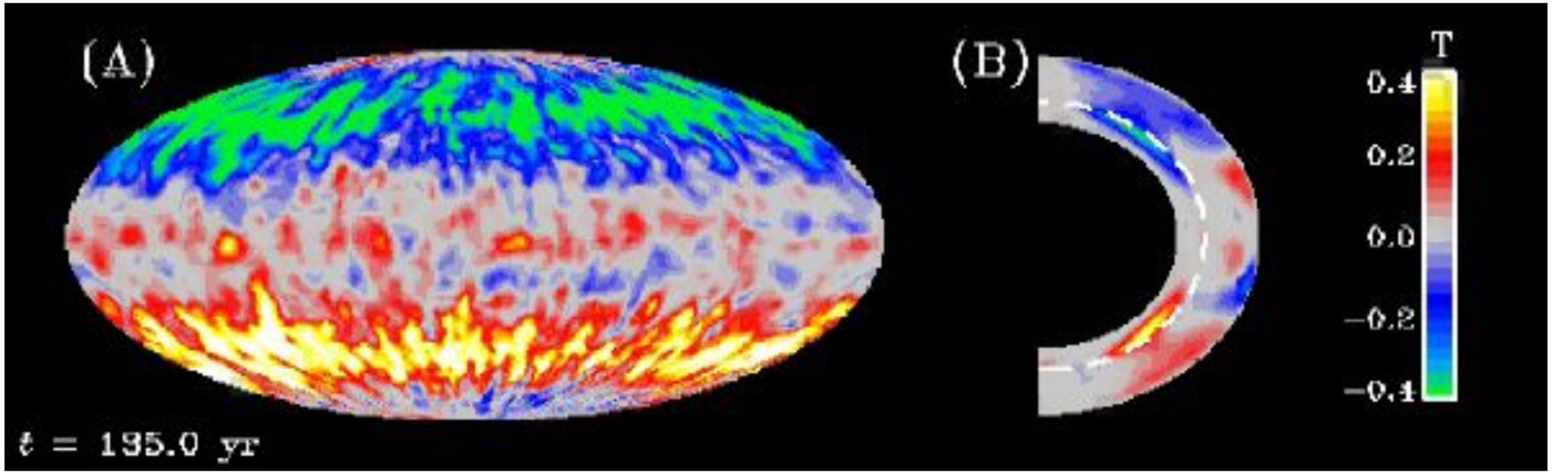


- Corrected P_{cyc} scales with P_{rot}^{-1}
- Not in disagreement with obs
- Not an $\alpha\Omega$ nor a BL dynamo

Spots in 3D models?

- ❑ 3D models produce magnetic cycles **without producing spots** and meridional circulation does not seem to set up the cycle period (Brown et al. 2011, Ghizaru et al. 2010, Nelson et al. 2013, Käpylä et al. 2013, Augustson et al. 2015, Hotta et al. 2016)

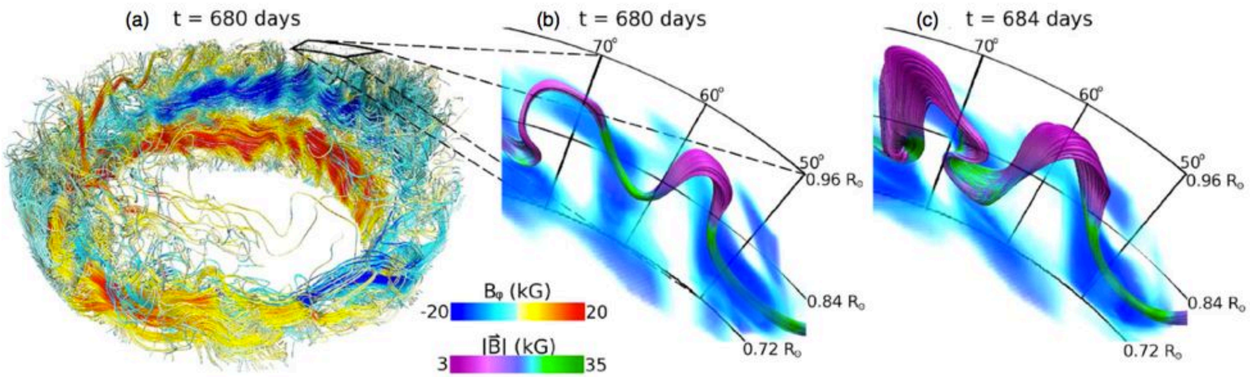
Eulag code



- ❑ Strong concentrations of toroidal field can still be built but buoyant structures do not make it to the top to produce spots!

Nelson et al. (2011, 2014)

ASH code



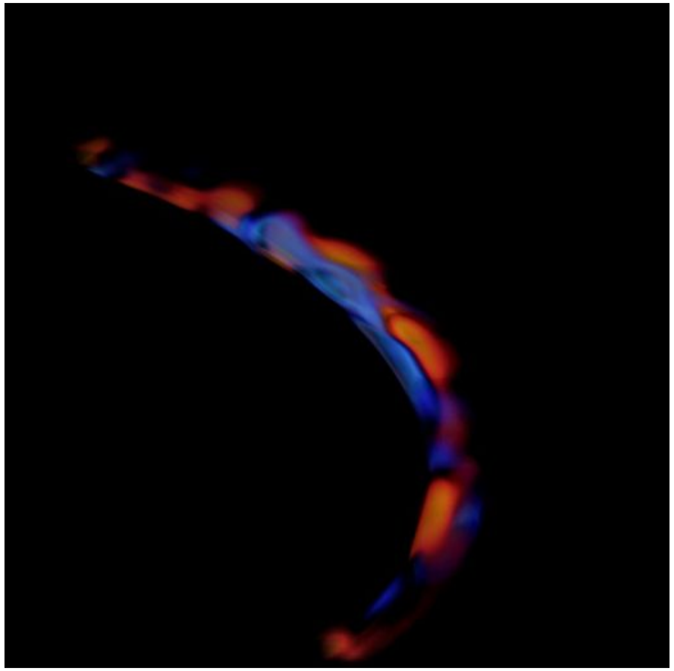
Simulation of buoyant loop rise and sunspots

- The buoyant rise has to be modeled independently:

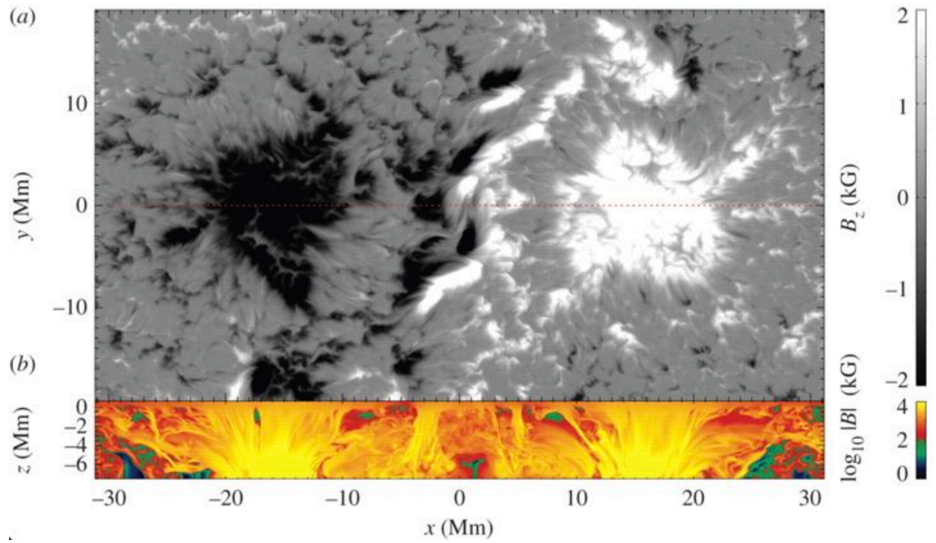
Toroidal flux tube introduced at the base of the CZ in a convective layer

ASH code

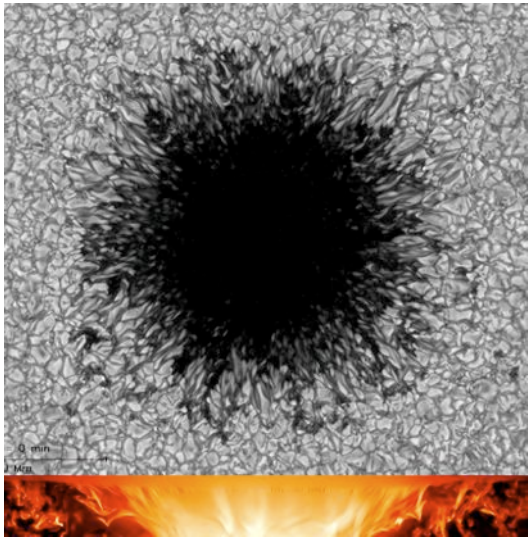
Jouve et al. 2013



- Or individual sunspots can be modeled in **radiative MHD codes (only upper CZ and atmosphere)**



Rempel et al. 2009, 2014

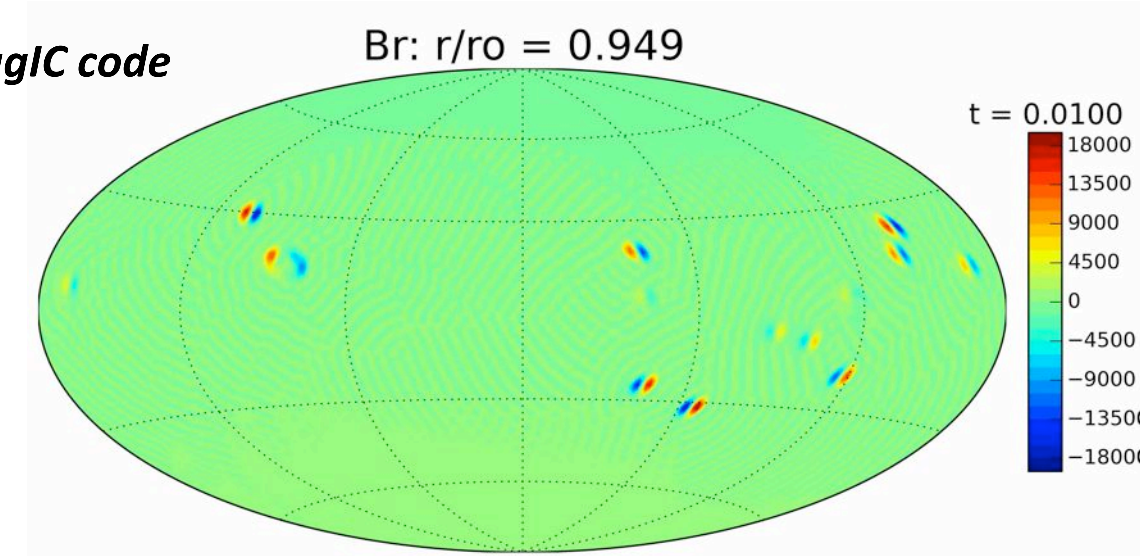


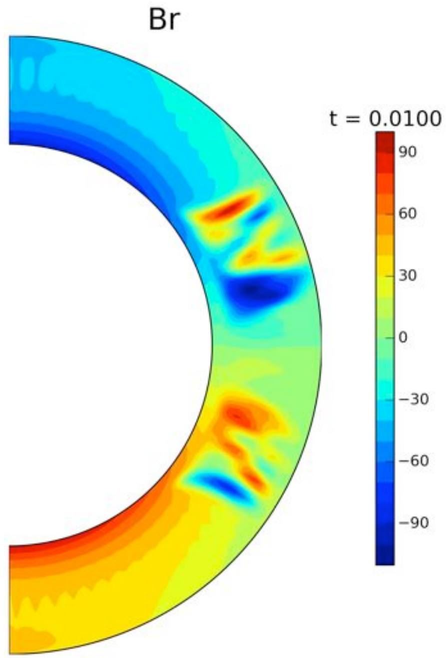
Muram code

3D kinematic models: combining approaches

- Mean-field dynamo models + 3D flux emergence and spot formation (Yeates & Munoz Jaramillo 2013, Miesch & Dikpati 2014, Miesch & Teweldebirhan 2016)

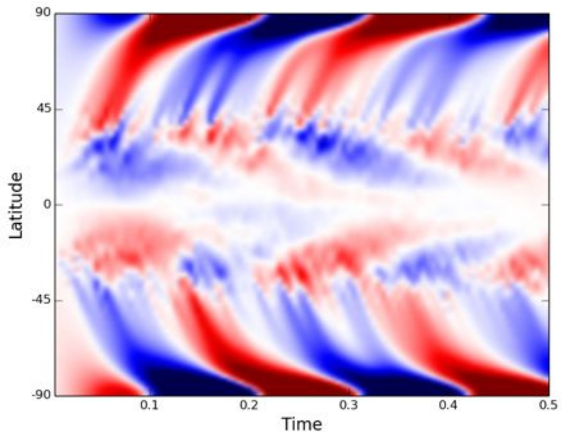
MagIC code

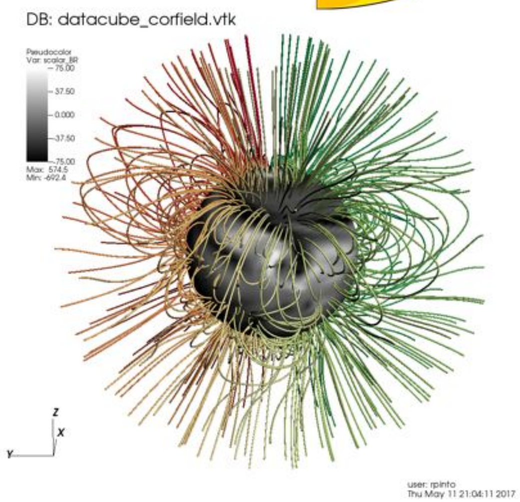




Kumar, Jouve, Pinto & Rouillard , 2018

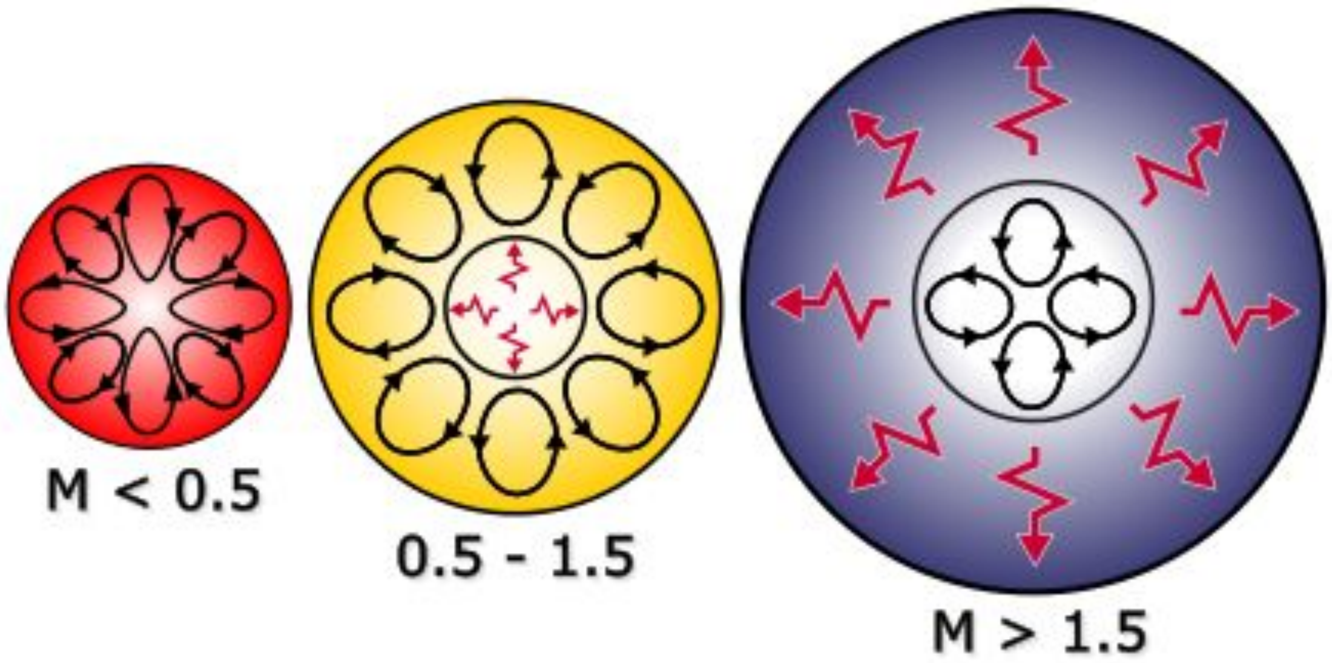
Self-consistent butterfly diagrams





Coronal field + wind solutions

Magnetism of more massive stars



❑ **In more massive stars (with radiative envelopes)**

- Only 5 to 10% are found to possess a strong magnetic field, they are Ap/Bp stars
- Magnetic field starts to be detected on non-Ap stars: much weaker and complex

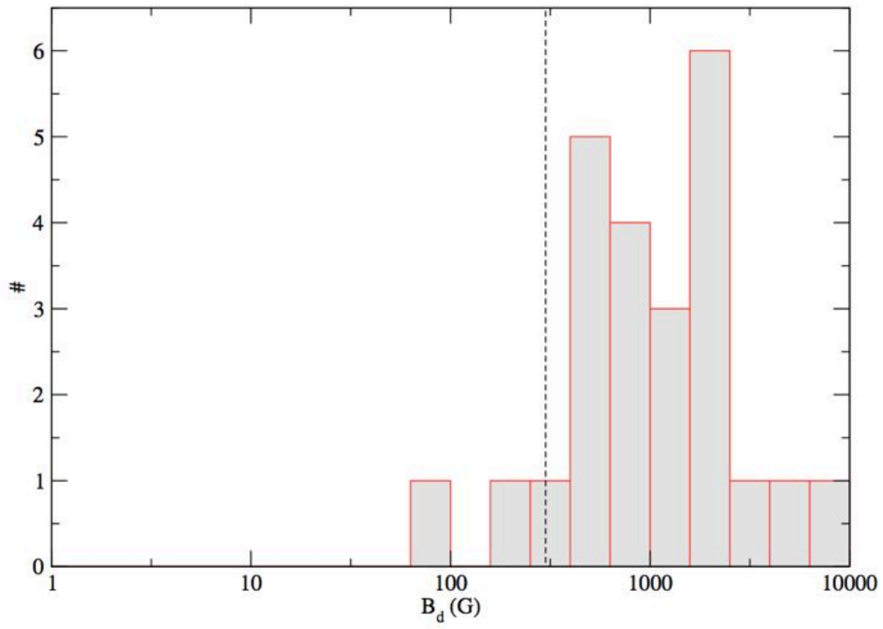
Ap/Bp magnetism

❑ **Origin of magnetic field in a star possessing a radiative envelope?**

Observations:

- Inclined dipole (Lüftinger et al 2010)
- Field intensity: either strong fields (B > 300 G) or no field (Aurière et al. 2007)
- No detection on large sample of Am or HgMn stars (Aurière et al. 2010)

Musicos + NARVAL



❑ **Why such a threshold?** (Aurière et al. 2007)

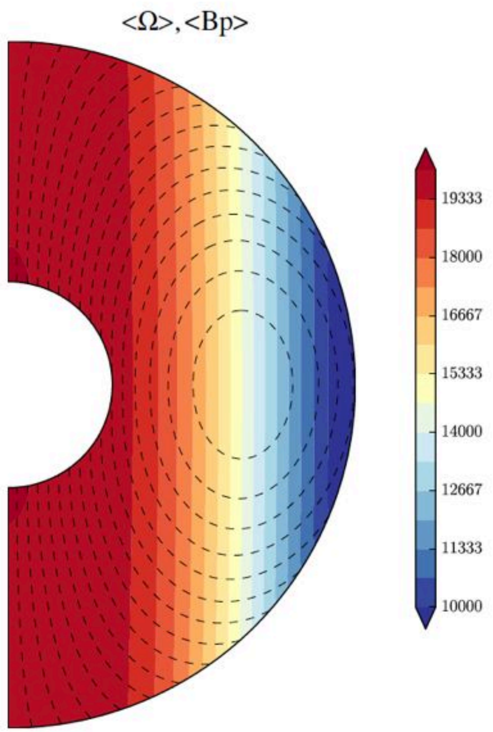
- Strong field ⟹ Differential rotation suppressed ⟹ Strong measured Bl
- Weak field ⟹ Toroidal field created by differential rotation and back-reacts:

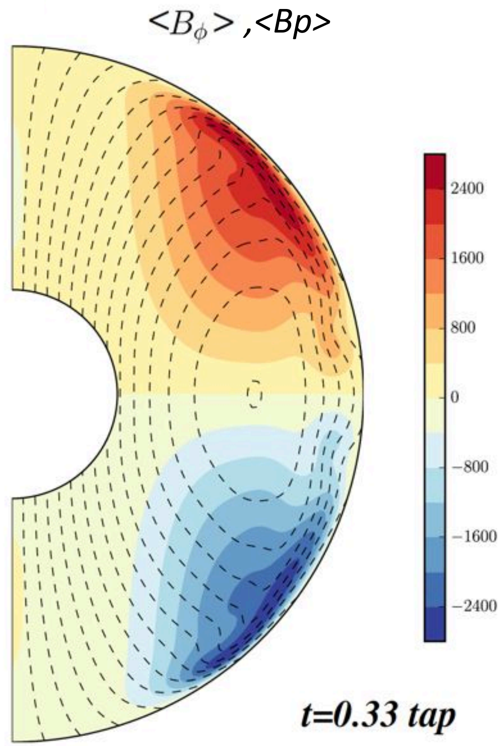
⟹ Structure dominated by toroidal field when $Max\left(\frac{B_\phi}{B_p}\right) \approx r\sin\theta\frac{\sqrt{4\pi\rho}\ \Omega}{B_p} \geq \alpha$

⟹ Possible instabilities for $Bp < Bc = r\sin\theta\sqrt{4\pi\rho}\Omega$

Numerical approach: 3D simulations
Initial value problem

MagIC code





- ❑ Initial conditions: poloidal field (Lu) wound-up by cylindrical differential rotation (Re)

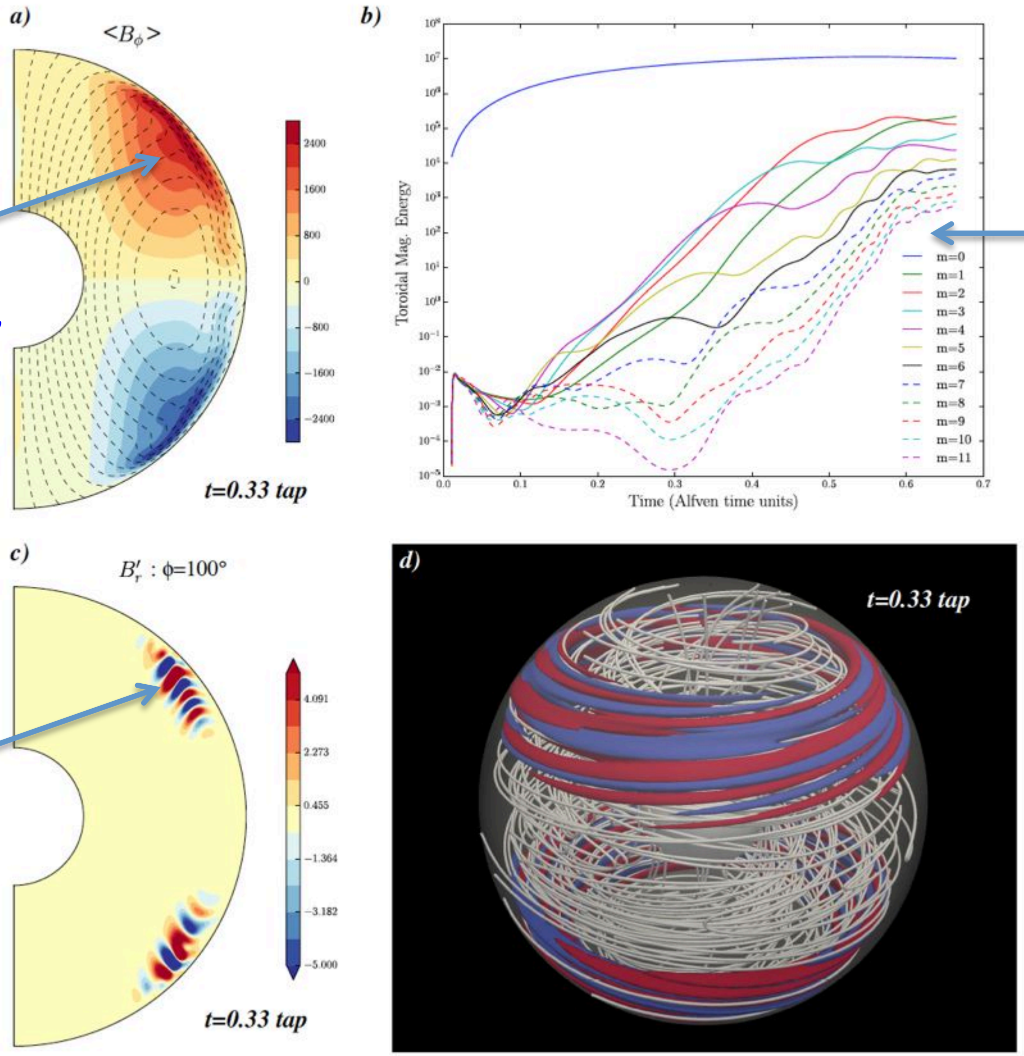
- ❑ A toroidal field is built which will then back-react on the differential rotation:
  - Is this configuration unstable?
  - Under which conditions is it triggered?
  - What are the consequences of this instability?

Evidence for an instability

- ❑ Typical case: Lu=60, Re=2 x 10^4: instability sets in around t=0.1 tap

MagIC code

- Strong toroidal field, antisymmetric, close to the surface



- Favored modes: m=4, 5 and 6

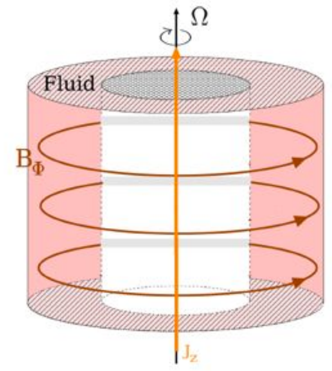
- Instability around the regions of strong toroidal field

Jouve et al., 2015

What is the nature of this instability?

- **Magneto-rotational instability:**

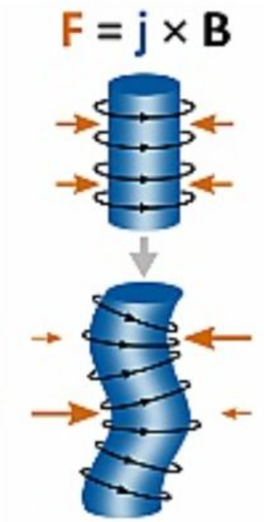
  - source of energy: kinetic energy of differential rotation (decreasing outward)
  - growth rate prop. to rotation rate and shear
  - high m's can be excited
  - necessitates weak field and strong differential rotation



Azimuthal MRI

- **Tayler instability:**

  - source of energy: magnetic energy
  - m=1 favored
  - growth rate prop. to Alfvén frequency
  - necessitates strong field and weak (differential) rotation

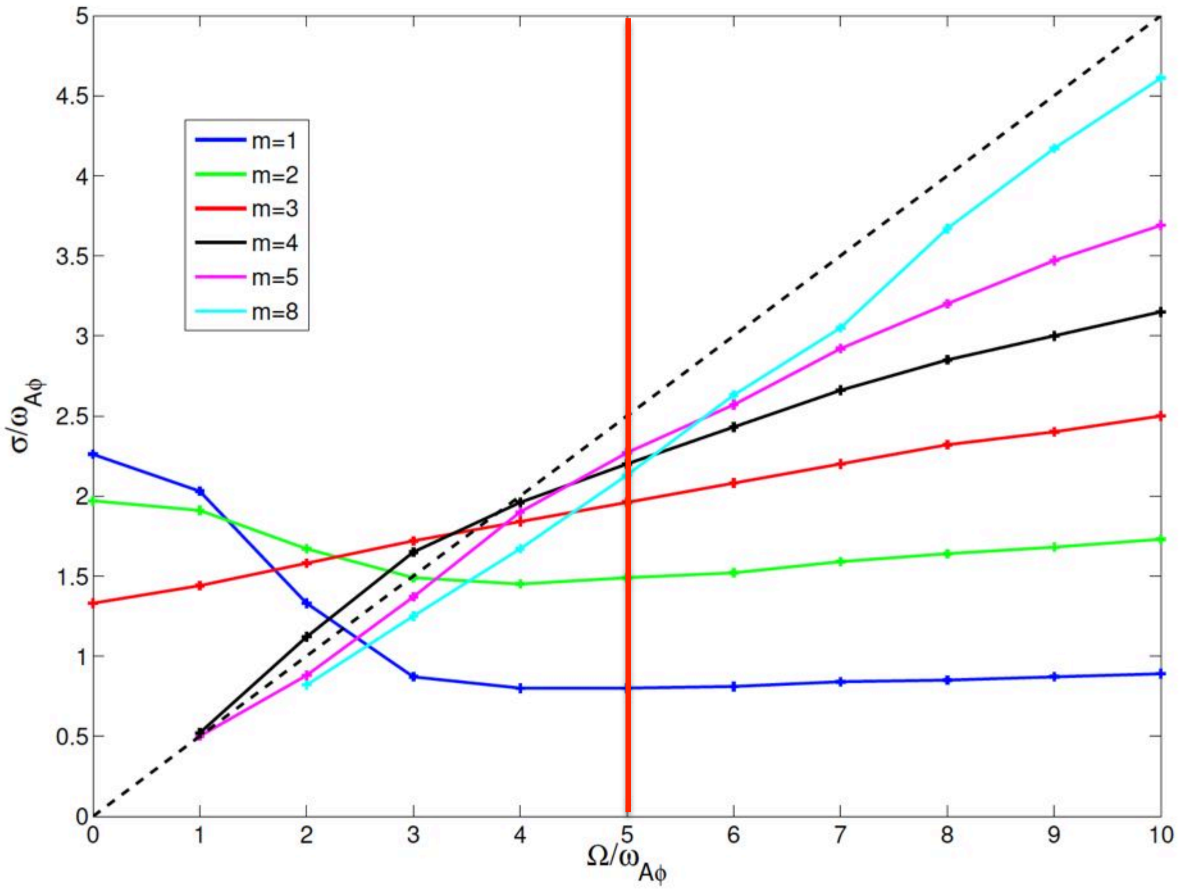


- **MRI vs TI:** importance of rotation rate (or shear) to toroidal Alfvén frequency ratio

What is the nature of this instability?

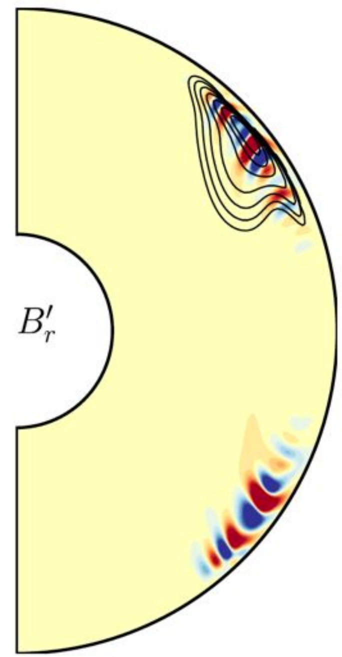
❑ MRI vs TI: importance of rotation rate to toroidal Alfvén frequency ratio: Ogilvie (2007)

$$\left[\omega^2 - \frac{m^2 B^2}{s^2} - 2\left(\frac{\Omega_0}{\omega_{A_{\phi 0}}}\right)^2 s\,\Omega\,\mathbf{e}_s \cdot \nabla\Omega + 2\,B\,\mathbf{e}_s \cdot \nabla\left(\frac{B}{s}\right)\right] \times \left[\omega^2 - \frac{m^2 B^2}{s^2}\right] = \left[2\left(\frac{\Omega_0}{\omega_{A_{\phi 0}}}\right)\omega\,\Omega + \frac{2mB^2}{s^2}\right]^2$$



In all our cases, the instability sets in when $\Omega / \omega_{A\varphi} \approx 5$

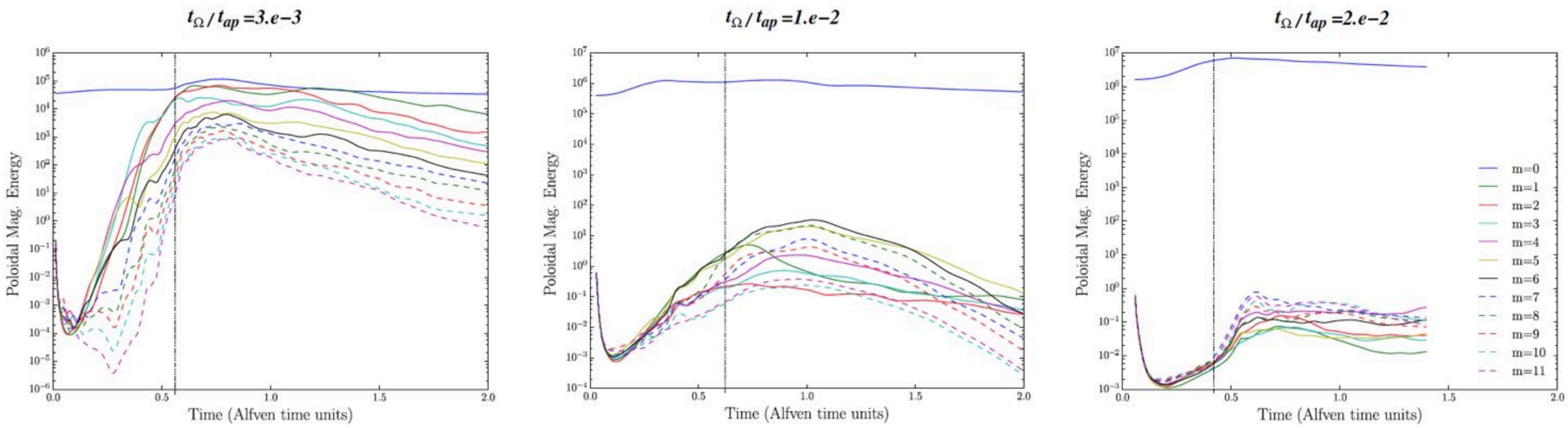
→ MRI regime



Consequences:
Application to A-type stars

- Background field evolves on poloidal Alfvén time scale t_{ap}
- Growth time of the MRI of the order of t_{Ω} (σ=q $\Omega/2$ with q around 1 here)

→ Stable and unstable cases distinguished by the ratio t_{Ω}/t_{ap}



Effects of stable stratification

- Additional parameters:
  - degree of stratification measured by N/Ω
  - Ratio of viscosity to thermal diffusivity measured by Pr
  - In stars, N/Ω is large (10^2-10^3) and Pr is small (10^{-6}-10^{-4})

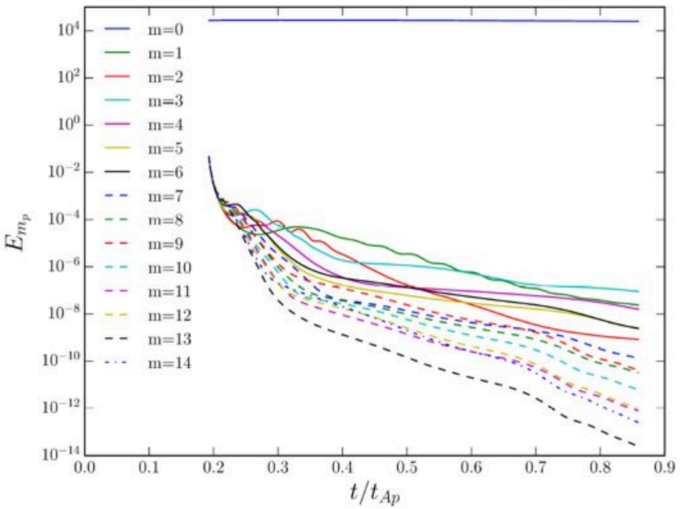
- We expect strong effects of stable stratification

- But a large thermal diffusion (small Pr) can help to reduce the effects of stratification

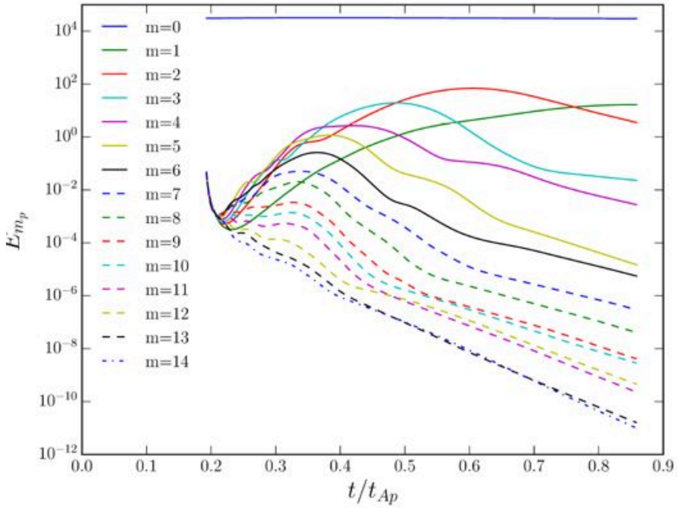
For N/Ω=5, the MRI:
- is lost for Pr=1
- recovered for $Pr=10^{-2}$

Gaurat et al., in prep.

Pr=1



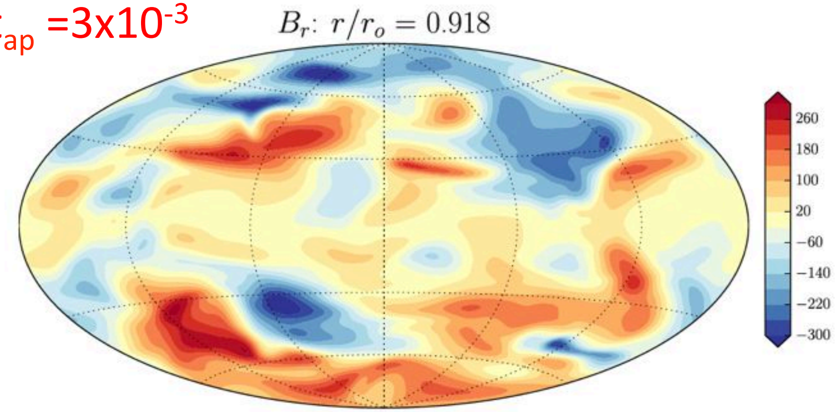
$Pr=10^{-2}$

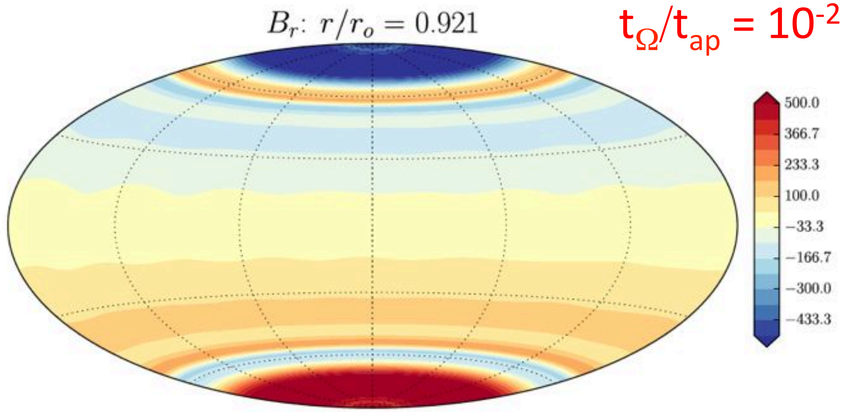


Application to A-type stars

- ❑ Surface radial field: non-axisymmetric VS axisymmetric
  - Unstratified cases

$t_{\Omega}/t_{ap} = 3\text{x}10^{-3}$

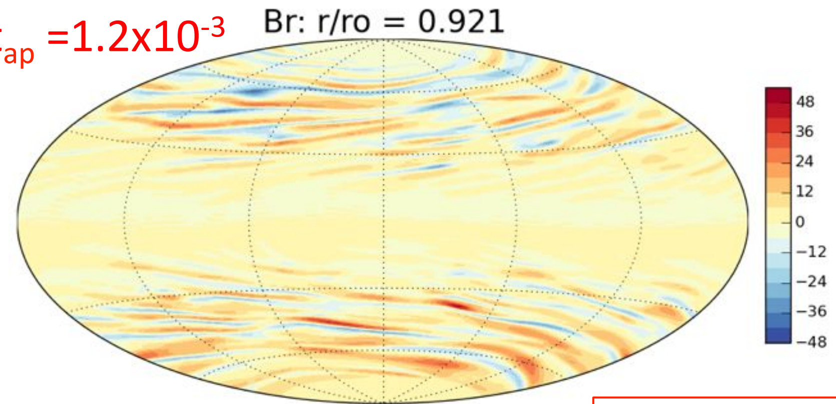


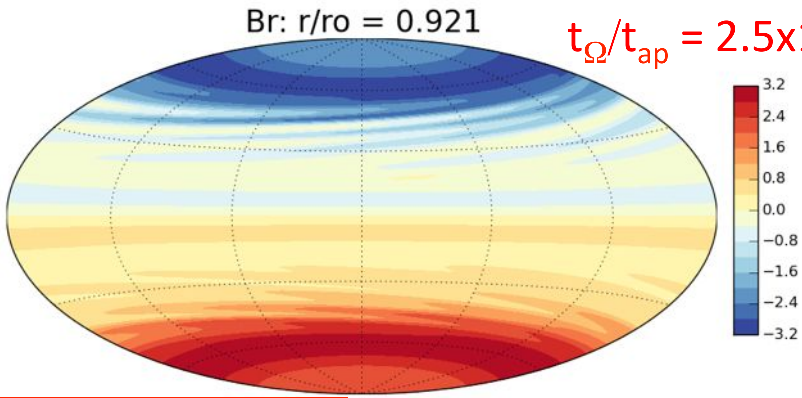


$t_{\Omega}/t_{ap} = 10^{-2}$

  - Stratified cases

$t_{\Omega}/t_{ap} = 1.2\text{x}10^{-3}$





$t_{\Omega}/t_{ap} = 2.5\text{x}10^{-3}$

- ❑ Estimate of threshold field:

$$B_{0_{crit}} = 10^{-2}\Omega_0 d \sqrt{\rho_0 \mu_0}$$

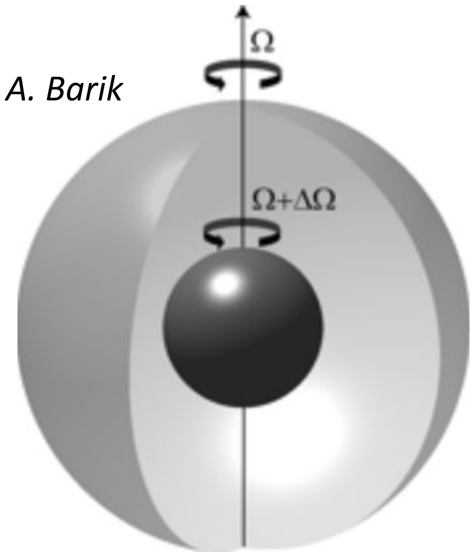
- ❑ Proportionality with rotation rate also seen in observations (Lignières et al. 2014)

Forced differential rotation

- Spherical Couette flow producing Stewartson layer and concentrated Bphi

Courtesy A. Barik

MagIC code

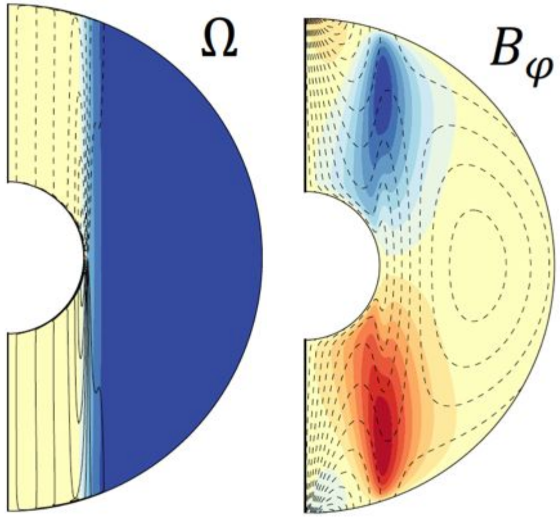


Meduri et al., subm.

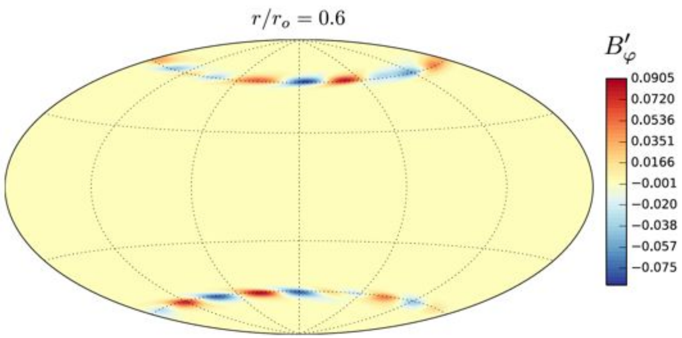
$$E = \frac{\nu}{\Omega d^2} = 10^{-5}$$

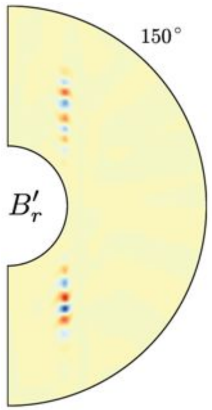
$$Ro = \frac{\Delta\Omega}{\Omega} = 0.03$$

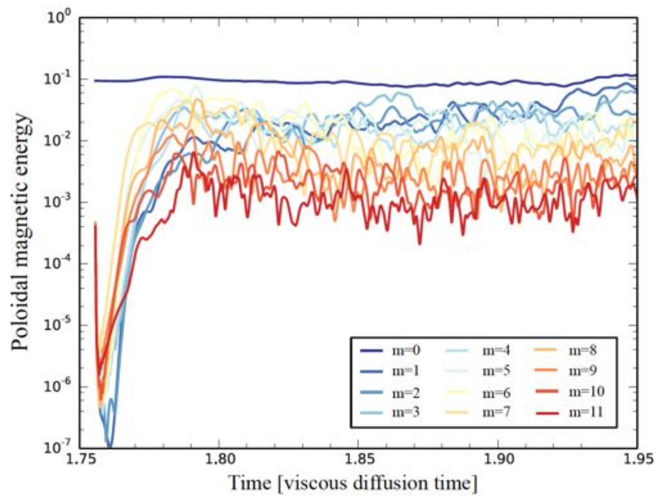
$$Rm = \frac{\Delta\Omega d^2}{\eta} = 2.10^4$$



- MRI and possible dynamo action?







More interaction with other communities?

- With the applied mathematicians

- Could we model a star from its deep interior to its atmosphere?

- For now, separate fields of research because (among other things) Mach number changes drastically

- Asymptotic-Preserving (AP) schemes: enable to design 1 scheme which deals with a set of equations and its asymptotic limit when a parameter goes to 0.

$$\begin{array}{ccc} P^{\varepsilon,h} & \xrightarrow{h\to 0} & P^{\varepsilon} \\ \Big\downarrow{\scriptstyle \varepsilon\to 0} & & \Big\downarrow{\scriptstyle \varepsilon\to 0} \\ P^{0,h} & \xrightarrow{h\to 0} & P^{0} \end{array}$$

Degond, lecture notes on AP schemes

More interaction with other communities?

- With the applied mathematicians

- The idea is that these schemes (which consist in impliciting well-chosen terms) will automatically adapt when going from ε small to ε=O(1).

- Uniform stability (independent of ε)

Compressible system $(P)_A^\epsilon$

$$\begin{cases} \partial_t \rho^\epsilon + \nabla \cdot (\rho^\epsilon \mathbf{u}^\epsilon) = 0 \\ \rho^\epsilon \left[\partial_t \mathbf{u}^\epsilon + (\mathbf{u}^\epsilon \cdot \nabla)\, \mathbf{u}^\epsilon\right] = -\frac{1}{\epsilon^2} \nabla p^\epsilon + \frac{1}{\epsilon^2} \rho^\epsilon g \\ \rho^\epsilon T^\epsilon \left[\partial_t S^\epsilon + (\mathbf{u}^\epsilon \cdot \nabla)\, S^\epsilon\right] = \epsilon^2 \nabla \cdot (\rho^\epsilon \nabla T^\epsilon) + \nabla \cdot (\rho^\epsilon T^\epsilon \nabla S^\epsilon), \end{cases}$$

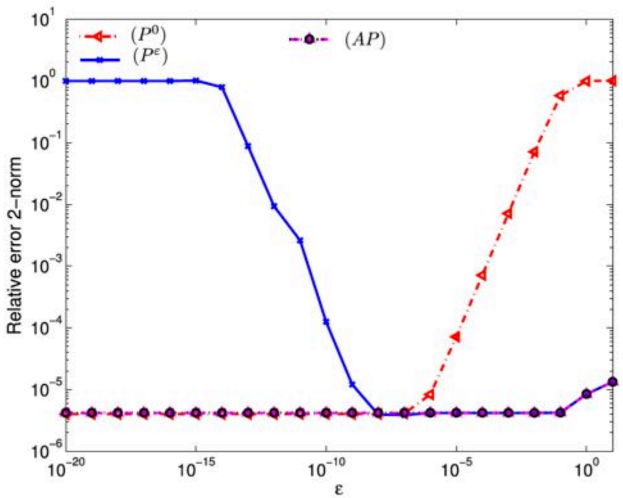
Anelastic limit ε -> 0 $(P)_A^0$

$$\begin{cases} \nabla \cdot (\bar{\rho}\, \bar{\mathbf{u}}) = 0 \\ \partial_z \bar{p} = \bar{\rho}\, g \\ \bar{\rho}\, [\partial_t \bar{\mathbf{u}} + (\bar{\mathbf{u}} \cdot \nabla) \bar{\mathbf{u}}] = -\nabla p' + \rho' g \\ \bar{\rho} \bar{T} \left[\partial_t S' + (\bar{\mathbf{u}} \cdot \nabla) S'\right] = \partial_z \left[\bar{\rho}\, \partial_z \bar{T}\right] + \nabla \cdot \left[\bar{\rho}\, \bar{T}\, \nabla S'\right]. \end{cases}$$

Mentrelli, 2018

- Example in plasma physics:

Degond & Deluzet, 2017, JCP



More interaction with other communities?

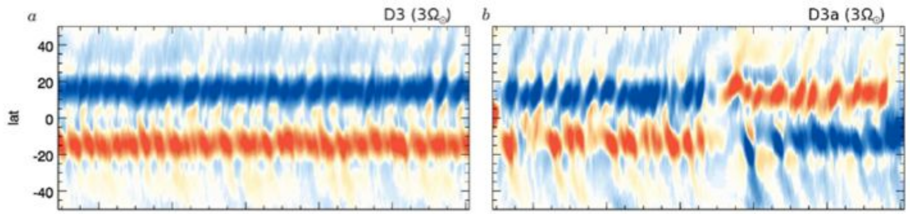
- With atmospheric scientists?

  - In general, large number of fluid problems have also been studied in the context of atmospsheric research (instabilities, convection, stably stratified flows,…)

  - Most codes used for stellar MHD perform DNS and can have entropy diffusion to deal with unresolved scales

  - Subgrid-scale modelling difficult when MHD problems are considered

Brown et al. 2011



Cyclic field when η decreased

  - « Implicit LES » used in Eulag code but no explicit transport coef so difficulty to compare with DNS (Strugarek et al. 2016: comparison between Eulag and ASH)

Structuring the community?

- Many different codes doing the same thing with similar numerical methods

  - MagIC, Parody, XSHELLS, ASH, (Rayleigh, Pencil, Dedalus, Eulag)
  - Only Rayleigh scales to a very hogh number of proc (> 100000)
  - Need to improve parallelisation to be competitive in France?
  - Need to gather more people around 1 particular code?

- Difficulty to get help from engineers because no permanent position for them

  - GPUs?
  - Help from engineers (close to researchers) are crucial (MagIC efficiency was improved by factor 2 thanks to B. Putigny who implemented SHTNS but… no position for him in IRAP!)