A general theory of thermo-compositional adiabatic and diabatic convection

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DEN/DANS/STMF: S. Kokh
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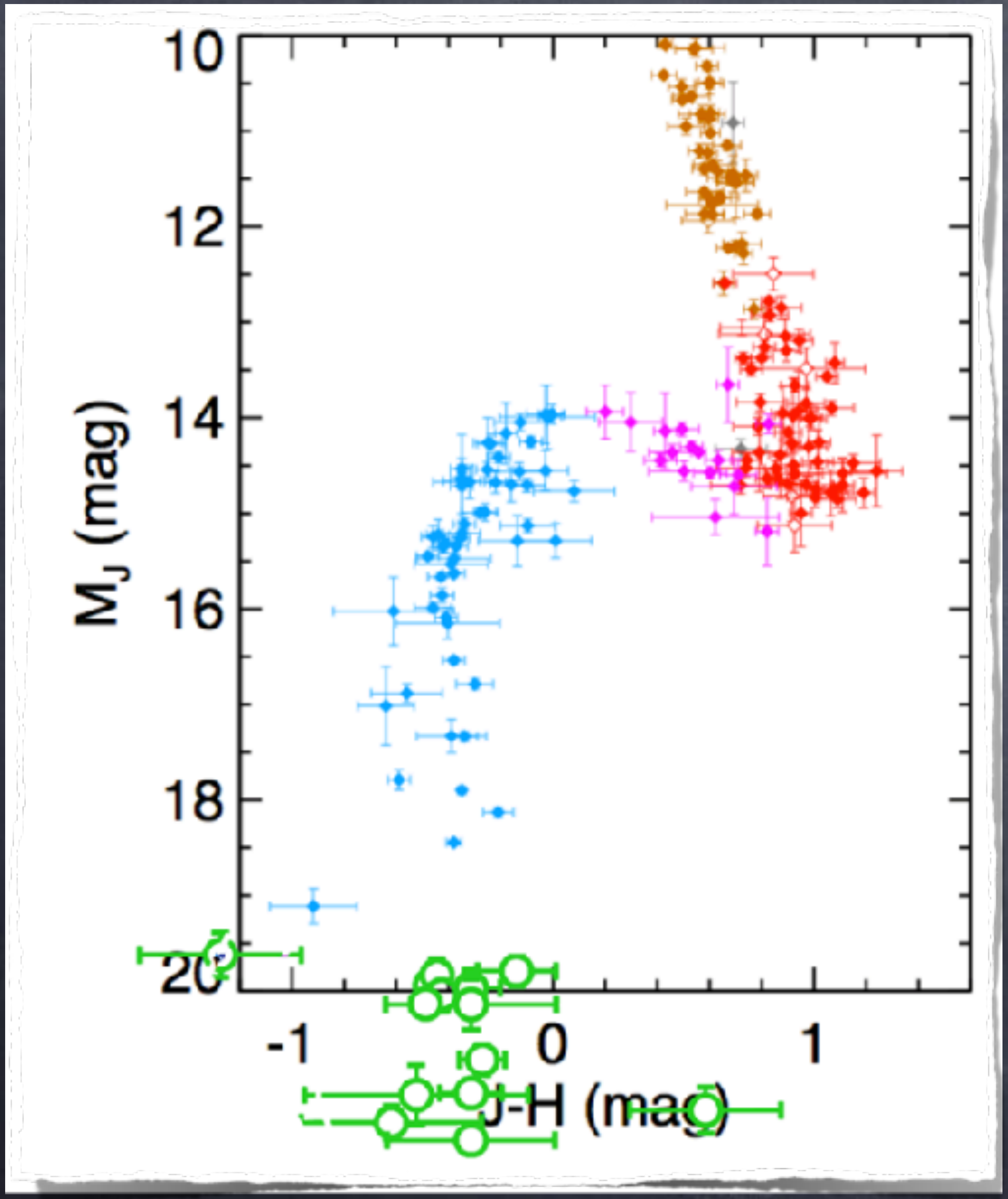
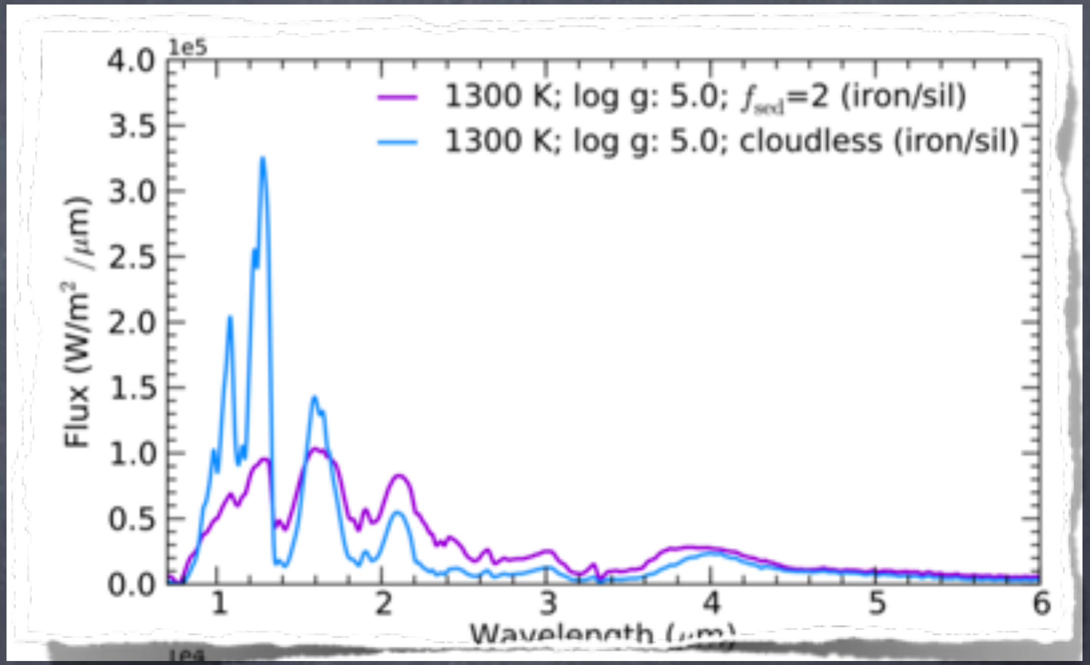
Exeter/Lyon: I. Baraffe, G. Chabrier, M. Phillips

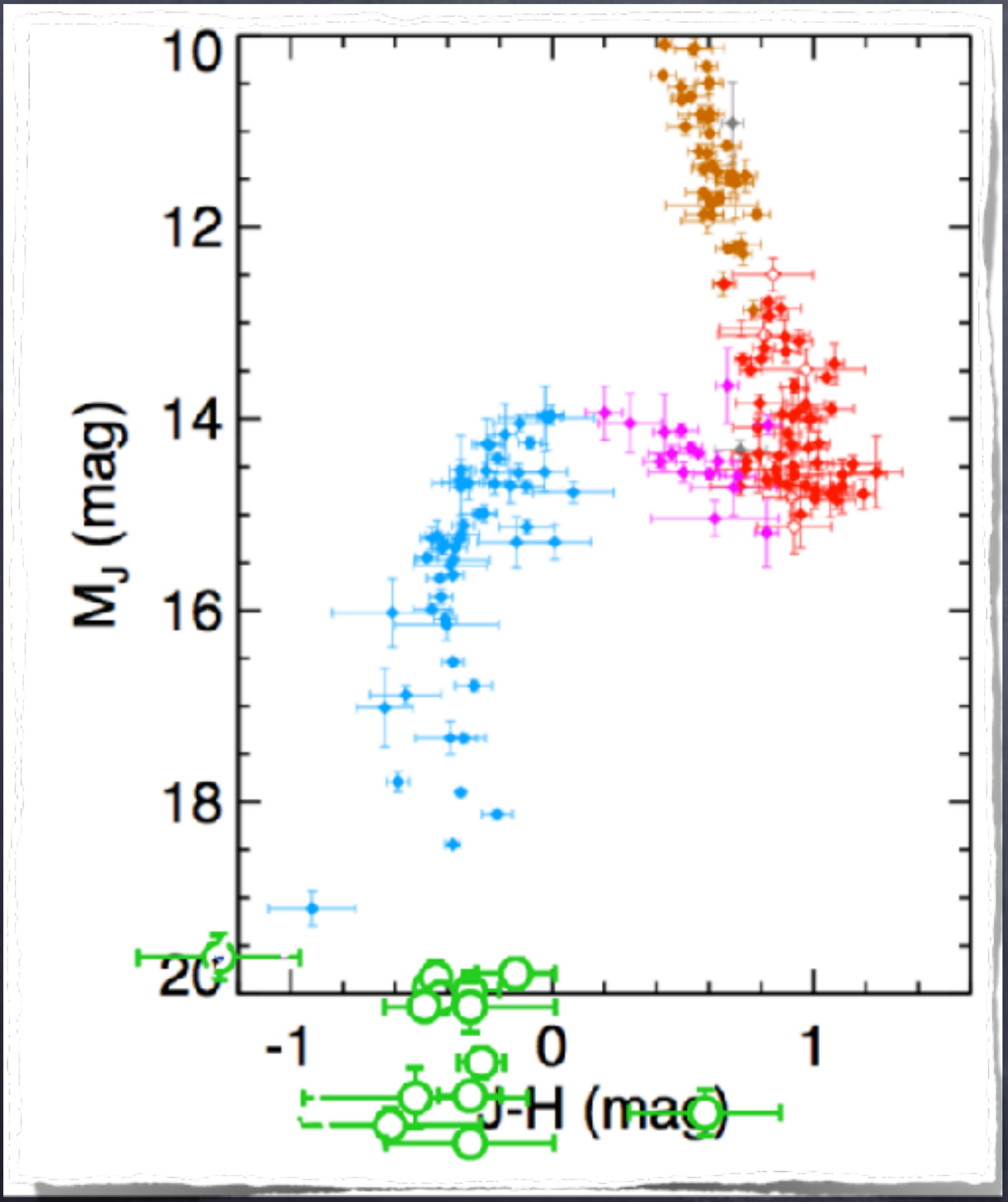
- Brown dwarfs spectral sequence:



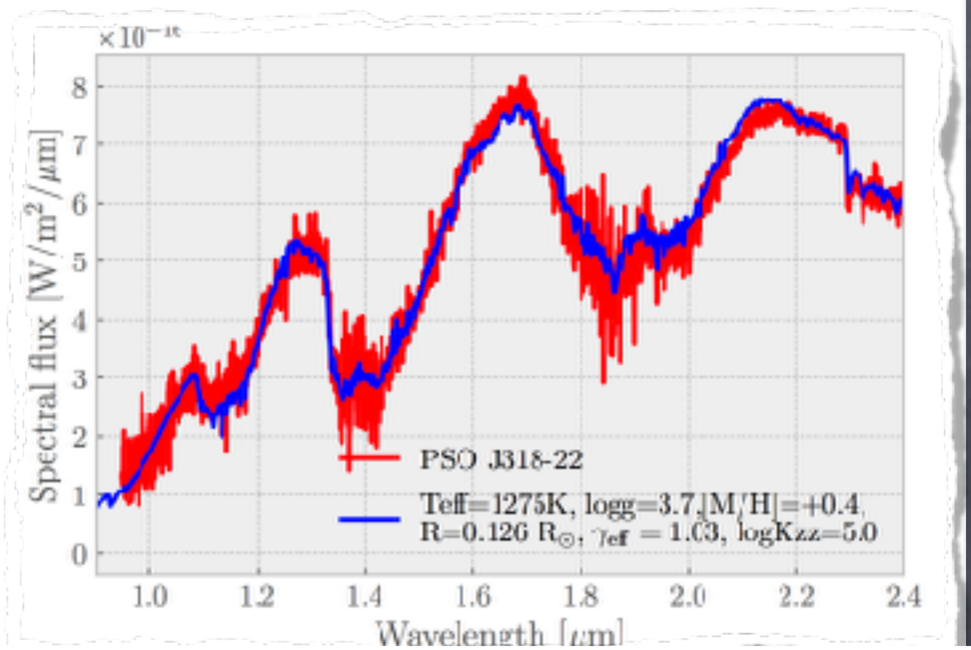
Clouds?

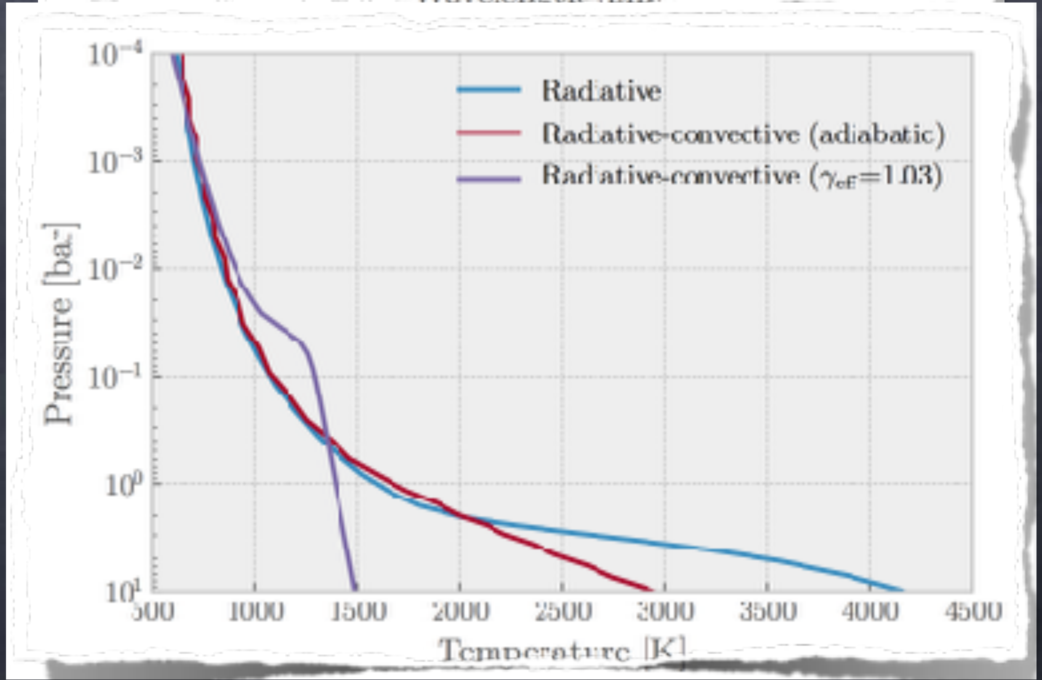


- Brown dwarfs spectral sequence:

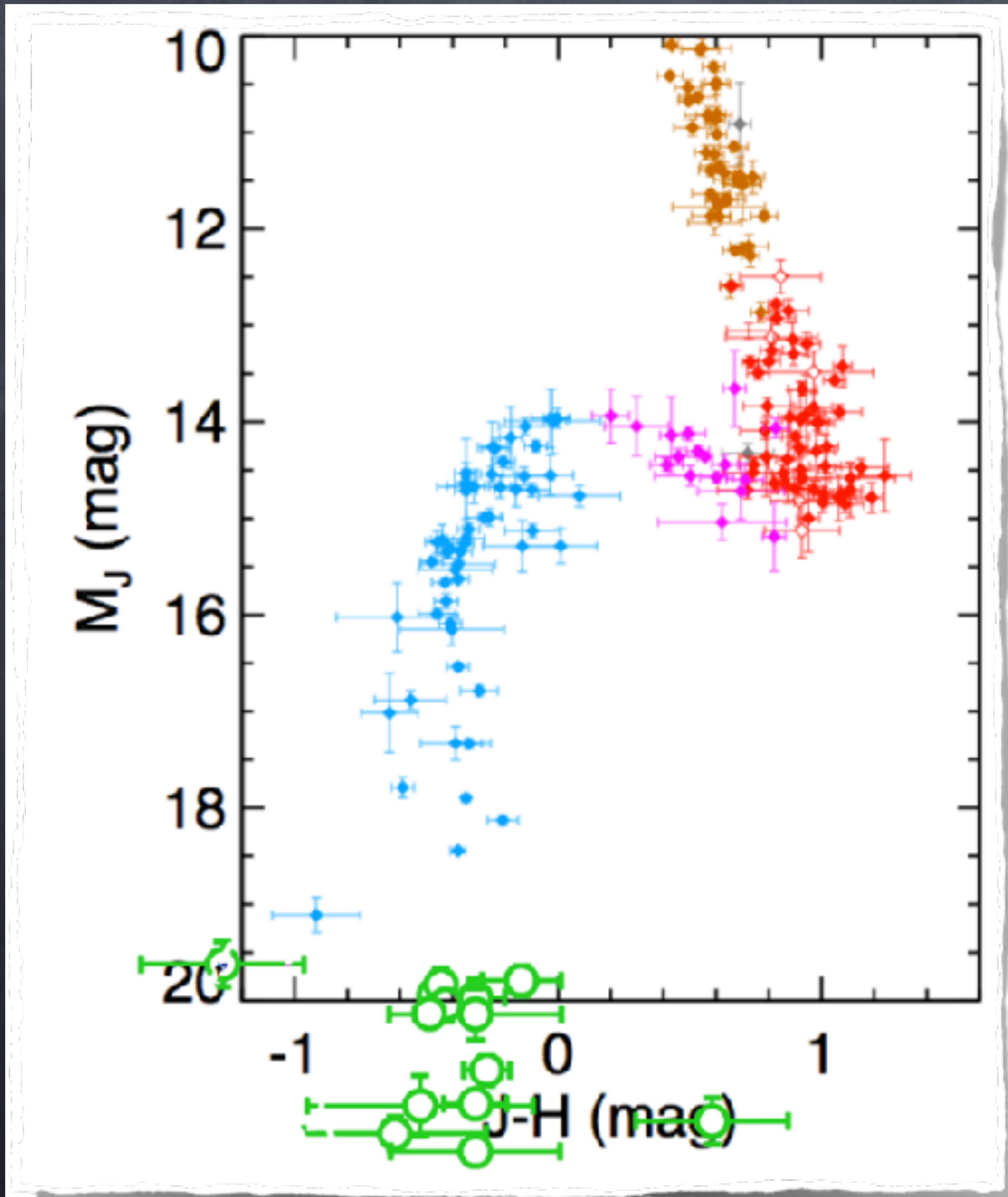


or reduced T gradient?

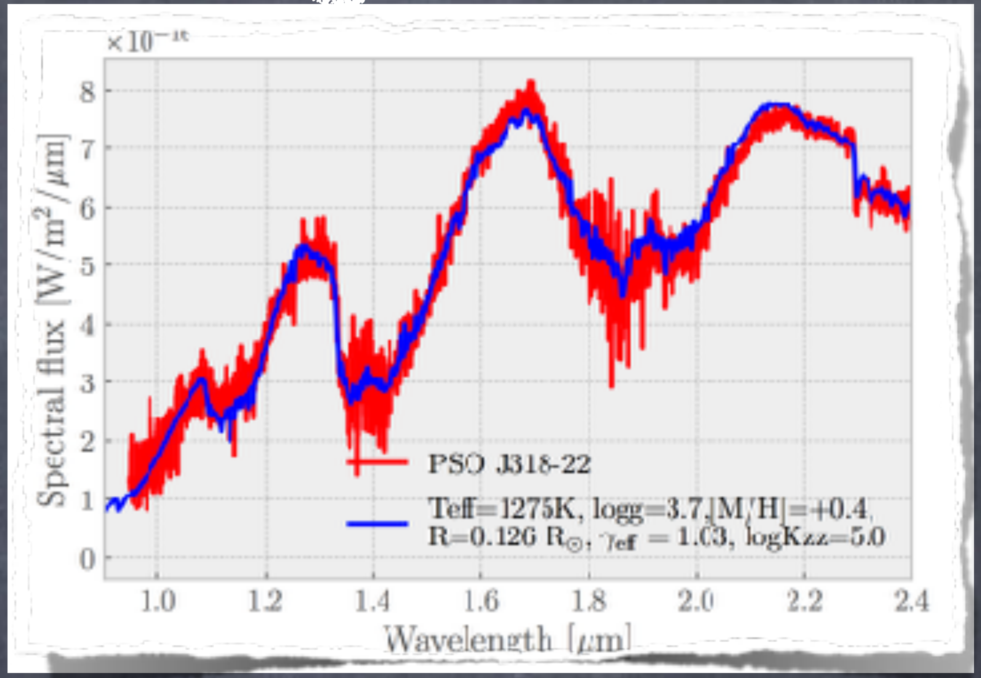




- Brown dwarfs spectral sequence:



or reduced T gradient?



Convection linked to CO/CH4 transition?

- Stratified compressible hydrodynamics

Numerical scheme, simulations and HPC implementation
-> Thomas Padioleau

with P. Kestener CEA/MdlS: HPC
with S. Kokh CEA/DEN: numerical scheme
and E. Audit CEA/MdlS

- Stratified compressible hydrodynamics

$$\frac{\partial \rho}{\partial t} + \overrightarrow{\nabla}\left(\rho \overrightarrow{u}\right) = 0$$

$$\frac{\partial \rho \overrightarrow{u}}{\partial t} + \overrightarrow{\nabla}\left(\rho \overrightarrow{u} \otimes \overrightarrow{u} + P\right) = \rho \overrightarrow{g}$$

$$\frac{\partial \rho \mathscr{E}}{\partial t} + \overrightarrow{\nabla}\left(\overrightarrow{u}(\rho \mathscr{E} + P)\right) = \rho c_p H(X, T)$$

$$\frac{\partial \rho X}{\partial t} + \overrightarrow{\nabla}\left(\rho X \overrightarrow{u}\right) = \rho R(X, T)$$

$$\mathscr{E} = e + \frac{1}{2}u^2 + \phi$$

$$\overrightarrow{g} = -\overrightarrow{\nabla}\phi$$

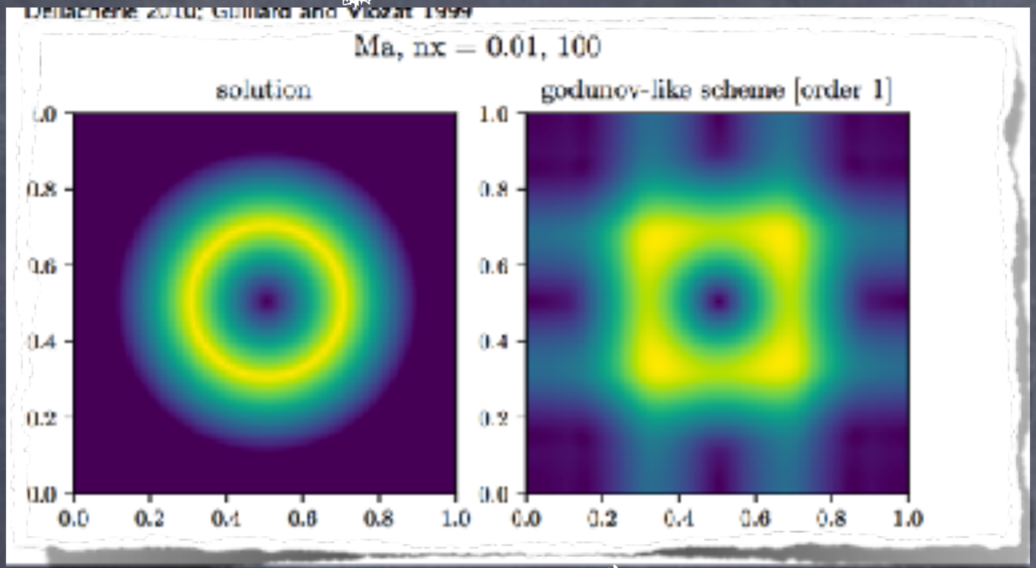
$$P = e(\gamma - 1) = \rho k_b T/\mu(X)$$

- Stratified compressible hydrodynamics
  - Compressibility/conservation
    - finite volume scheme
    - co-localised variables
  - Problems
    - poor accuracy at low Mach
    - small timestep (dt=dx/c)
    - poor hydrostatic balance

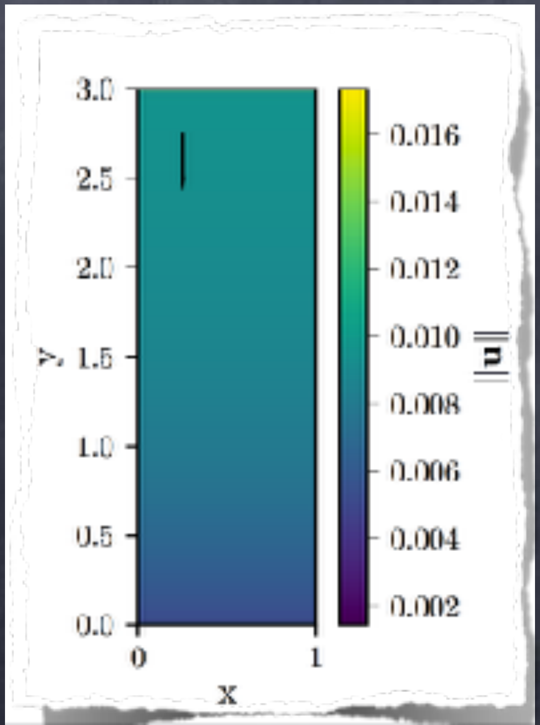
- Stratified compressible hydrodynamics

  - Problems

    - poor accuracy at low Mach



    - poor hydrostatic balance



- Stratified compressible hydrodynamics
  - All-regime solver: full scheme

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_x)}{\partial x} = 0$$

$$\frac{\partial \rho u_x}{\partial t} + \frac{\partial(\rho u_x^2 + P)}{\partial x} = 0$$

$$\frac{\partial \rho \mathcal{E}}{\partial t} + \frac{\partial((\rho \mathcal{E} + P)u_x)}{\partial x} = 0$$

- Stratified compressible hydrodynamics
  - All-regime solver: full scheme

$$\frac{\partial \rho}{\partial t} + \rho \frac{\partial u_x}{\partial x} + u_x \frac{\partial \rho}{\partial x} = 0$$

$$\frac{\partial \rho u_x}{\partial t} + \rho u_x \frac{\partial u_x}{\partial x} + \frac{\partial P}{\partial x} + u_x \frac{\partial \rho u_x}{\partial x} = 0$$

$$\frac{\partial \rho \mathcal{E}}{\partial t} + \rho \mathcal{E} \frac{\partial u_x}{\partial x} + \frac{\partial P u_x}{\partial x} + u_x \frac{\partial \rho \mathcal{E}}{\partial x} = 0$$

- Stratified compressible hydrodynamics
  - All-regime solver: acoustic step

$$\frac{\partial \rho}{\partial t} + \rho \frac{\partial u_x}{\partial x} = 0$$

$$\frac{\partial \rho u_x}{\partial t} + \rho u_x \frac{\partial u_x}{\partial x} + \frac{\partial P}{\partial x} = 0$$

$$\frac{\partial \rho \mathcal{E}}{\partial t} + \rho \mathcal{E} \frac{\partial u_x}{\partial x} + \frac{\partial P u_x}{\partial x} = 0$$

- Stratified compressible hydrodynamics
  - All-regime solver: acoustic step

$$\frac{\partial \tau}{\partial t} - \frac{\partial u_x}{\partial m} = 0$$

$$\frac{\partial u_x}{\partial t} + \frac{\partial P}{\partial m} = 0$$

$$\frac{\partial \mathcal{E}}{\partial t} + \frac{\partial P u_x}{\partial m} = 0$$

$$\tau = \frac{1}{\rho}$$

$$dm = \rho dx$$

- Stratified compressible hydrodynamics
  - All-regime solver: acoustic step

$$\frac{\partial \tau}{\partial t} - \frac{\partial u_x}{\partial m} = 0$$

$$\frac{\partial u_x}{\partial t} + \frac{\partial \Pi}{\partial m} = 0$$

$$\frac{\partial \mathcal{E}}{\partial t} + \frac{\partial \Pi u_x}{\partial m} = 0$$

$$\frac{\partial \Pi}{\partial t} + a^2 \frac{\partial u_x}{\partial m} = 0$$

$$\tau = \frac{1}{\rho}$$

$$dm = \rho dx$$

$$a = \rho c_s$$

$$\Pi^{n+1-} = p(\tau^{n+1-}, \mathcal{E}^{n+1-}, u_x^{n+1-})$$

- Stratified compressible hydrodynamics
  - All-regime solver: acoustic step

$$U^{\star}_{i+1/2} = \frac{u_{x,i} + u_{x,i+1}}{2} - \frac{1}{2a}\left(\Pi_{i+1} - \Pi_i\right)$$

$$\Pi^{\star}_{i+1/2} = \frac{\Pi_i + \Pi_{i+1}}{2} - \frac{a}{2}\left(u_{x,i+1} - u_{x,i}\right)$$

- Stratified compressible hydrodynamics
  - All-regime solver: acoustic step

$$U^\star_{i+1/2} = \frac{u_{x,i} + u_{x,i+1}}{2} - \frac{1}{2a}\left(\Pi_{i+1} - \Pi_i\right)$$

$$\Pi^\star_{i+1/2} = \frac{\Pi_i + \Pi_{i+1}}{2} - \frac{a}{2}\left(u_{x,i+1} - u_{x,i}\right)$$

$$\frac{\tau^{n+1-} - \tau^n}{\Delta t} - \frac{U^\star_{i+1/2} - U^\star_{i-1/2}}{\Delta m} = 0$$

$$\frac{u_x^{n+1-} - u_x^n}{\Delta t} + \frac{\Pi^\star_{i+1/2} - \Pi^\star_{i-1/2}}{\Delta m} = 0$$

$$\frac{\mathscr{E}^{n+1-} - \mathscr{E}^n}{\Delta t} + \frac{U^\star_{i+1/2}\Pi^\star_{i+1/2} - U^\star_{i-1/2}\Pi^\star_{i-1/2}}{\Delta m} = 0$$

- Stratified compressible hydrodynamics
  - All-regime solver: transport step

$$\frac{\partial \rho}{\partial t} + u_x \frac{\partial \rho}{\partial x} = 0$$

$$\frac{\partial \rho u_x}{\partial t} + u_x \frac{\partial \rho u_x}{\partial x} = 0$$

$$\frac{\partial \rho \mathcal{E}}{\partial t} + u_x \frac{\partial \rho \mathcal{E}}{\partial x} = 0$$

- Stratified compressible hydrodynamics
  - All-regime solver: transport step

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_x}{\partial x} - \rho \frac{\partial u_x}{\partial x} = 0$$

$$\frac{\partial \rho u_x}{\partial t} + \frac{\partial \rho u_x^2}{\partial x} - \rho u_x \frac{\partial u_x}{\partial x} = 0$$

$$\frac{\partial \rho \mathcal{E}}{\partial t} + \frac{\partial \rho \mathcal{E} u_x}{\partial x} - \rho \mathcal{E} \frac{\partial u_x}{\partial x} = 0$$

- Stratified compressible hydrodynamics
  - All-regime solver: transport step

$$\frac{\rho^{n+1} - \rho^{n+1-}}{\Delta t} + \frac{[\rho^{n+1-} U^\star]}{\Delta x} - \rho^{n+1-} \frac{[U^\star]}{\Delta x} = 0$$

$$\frac{(\rho u_x)^{n+1} - (\rho u_x)^{n+1-}}{\Delta t} + \frac{[(\rho u_x)^{n+1-} U^\star]}{\Delta x} - (\rho u_x)^{n+1-} \frac{[U^\star]}{\Delta x} = 0$$

$$\frac{(\rho \mathscr{E})^{n+1} - (\rho \mathscr{E})^{n+1-}}{\Delta t} + \frac{[(\rho \mathscr{E})^{n+1-} U^\star]}{\Delta x} - (\rho \mathscr{E})^{n+1-} \frac{[U^\star]}{\Delta x} = 0$$

- Stratified compressible hydrodynamics
  - All-regime solver: full scheme

$$\frac{\rho^{n+1}-\rho^n}{\Delta t}+\frac{[\rho^{n+1-}U^\star]}{\Delta x}=0$$

$$\frac{(\rho u_x)^{n+1}-(\rho u_x)^n}{\Delta t}+\frac{[(\rho u_x)^{n+1-}U^\star+\Pi^\star]}{\Delta x}=0$$

$$\frac{(\rho \mathscr{E})^{n+1}-(\rho \mathscr{E})^n}{\Delta t}+\frac{[(\rho \mathscr{E})^{n+1-}U^\star+\Pi^\star U^\star]}{\Delta x}=0$$

$$U^\star_{i+1/2}=\frac{u_{x,i}+u_{x,i+1}}{2}-\frac{1}{2a}\left(\Pi_{i+1}-\Pi_i\right)$$

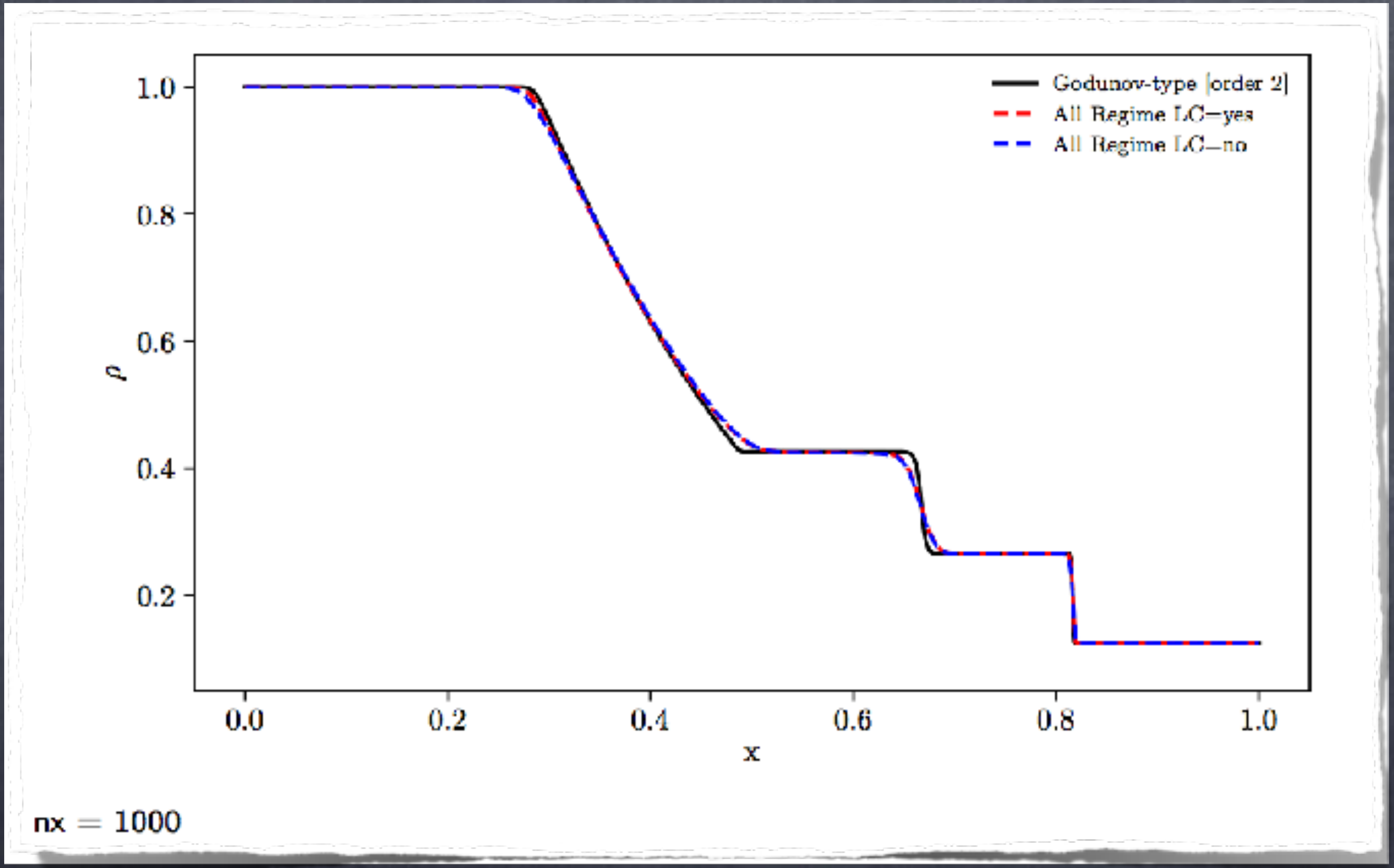
$$\Pi^\star_{i+1/2}=\frac{\Pi_i+\Pi_{i+1}}{2}-\frac{a}{2}\left(u_{x,i+1}-u_{x,i}\right)$$

- Stratified compressible hydrodynamics
  - All-regime solver: full scheme
    - Explicit/Explicit scheme
    - Implicit/Explicit scheme
    - Low Mach correction

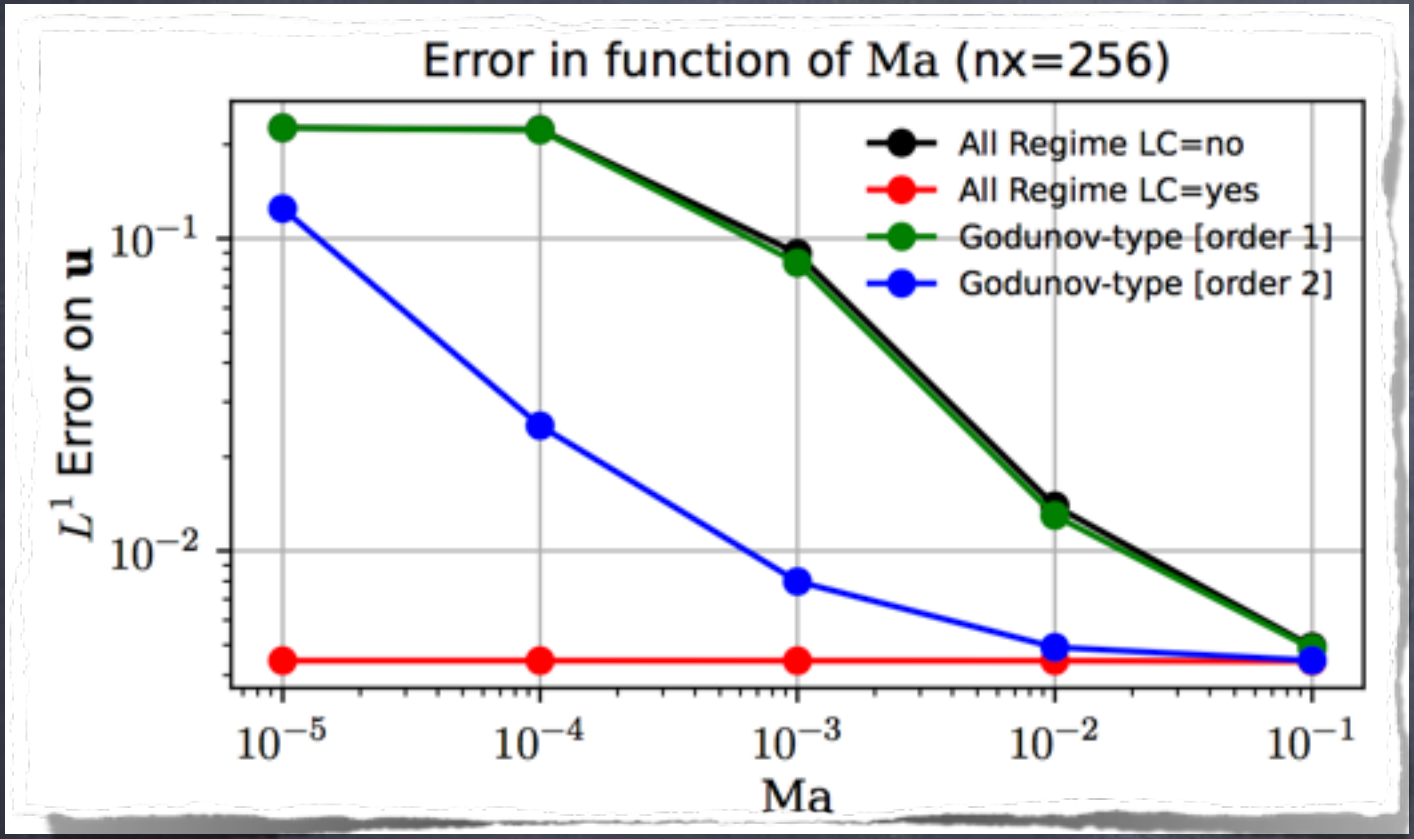
$$U^{\star}_{i+1/2} = \frac{u_{x,i} + u_{x,i+1}}{2} - \frac{1}{2a}\left(\Pi_{i+1} - \Pi_i\right)$$

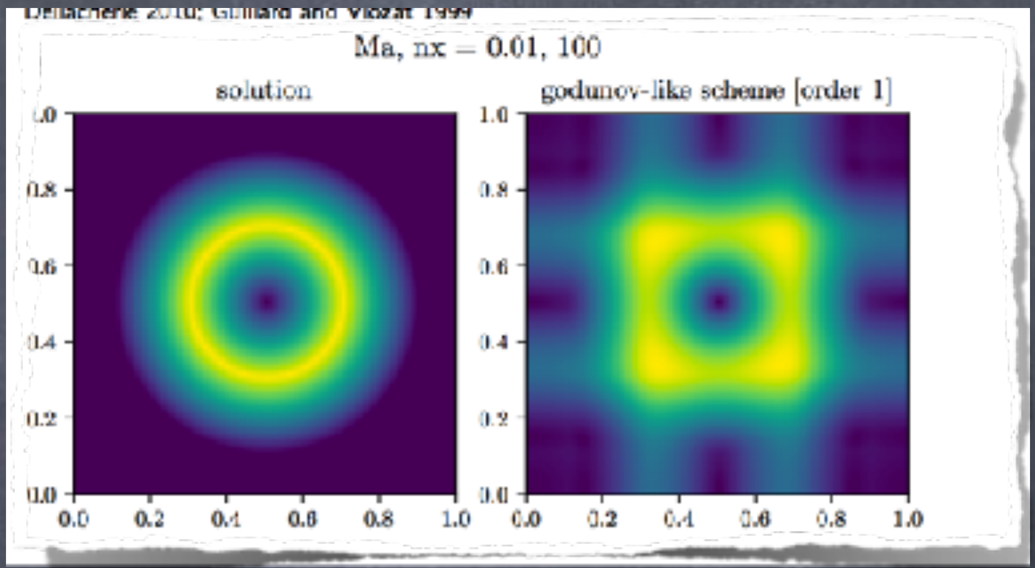
$$\Pi^{\star}_{i+1/2} = \frac{\Pi_i + \Pi_{i+1}}{2} - \frac{a}{2}\left(u_{x,i+1} - u_{x,i}\right)$$

- Stratified compressible hydrodynamics
  - All-regime solver: full scheme
    - conservative scheme: Sod test



- Stratified compressible hydrodynamics
  - All-regime solver: full scheme
    - Low Mach correction: Gresho vortex





- Stratified compressible hydrodynamics
  - All-regime solver with gravity
    - hydrostatic balance at cell centre

$$\frac{(\rho u_x)^{n+1} - (\rho u_x)^n}{\Delta t} + \frac{[(\rho u_x)^{n+1-}U^\star + \Pi^\star]}{\Delta x} = -\rho g$$

if there is initially no velocity

$$\frac{\Pi_i + \Pi_{i+1}}{2} - \frac{\Pi_i + \Pi_{i-1}}{2} = -\rho_i g \Delta x$$

$$u_x^{n+1} = O(\Delta x), \quad \frac{\partial P}{\partial x} = -\rho g + O(\Delta x)$$

- Stratified compressible hydrodynamics
  - All-regime solver with gravity
    - hydrostatic balance at interface

$$\frac{(\rho u_x)^{n+1} - (\rho u_x)^n}{\Delta t} + \frac{[(\rho u_x)^{n+1-}U^\star + \Pi^\star]}{\Delta x} = -\frac{1}{2}\left(\frac{\rho_i + \rho_{i+1}}{2} + \frac{\rho_i + \rho_{i-1}}{2}\right) g$$

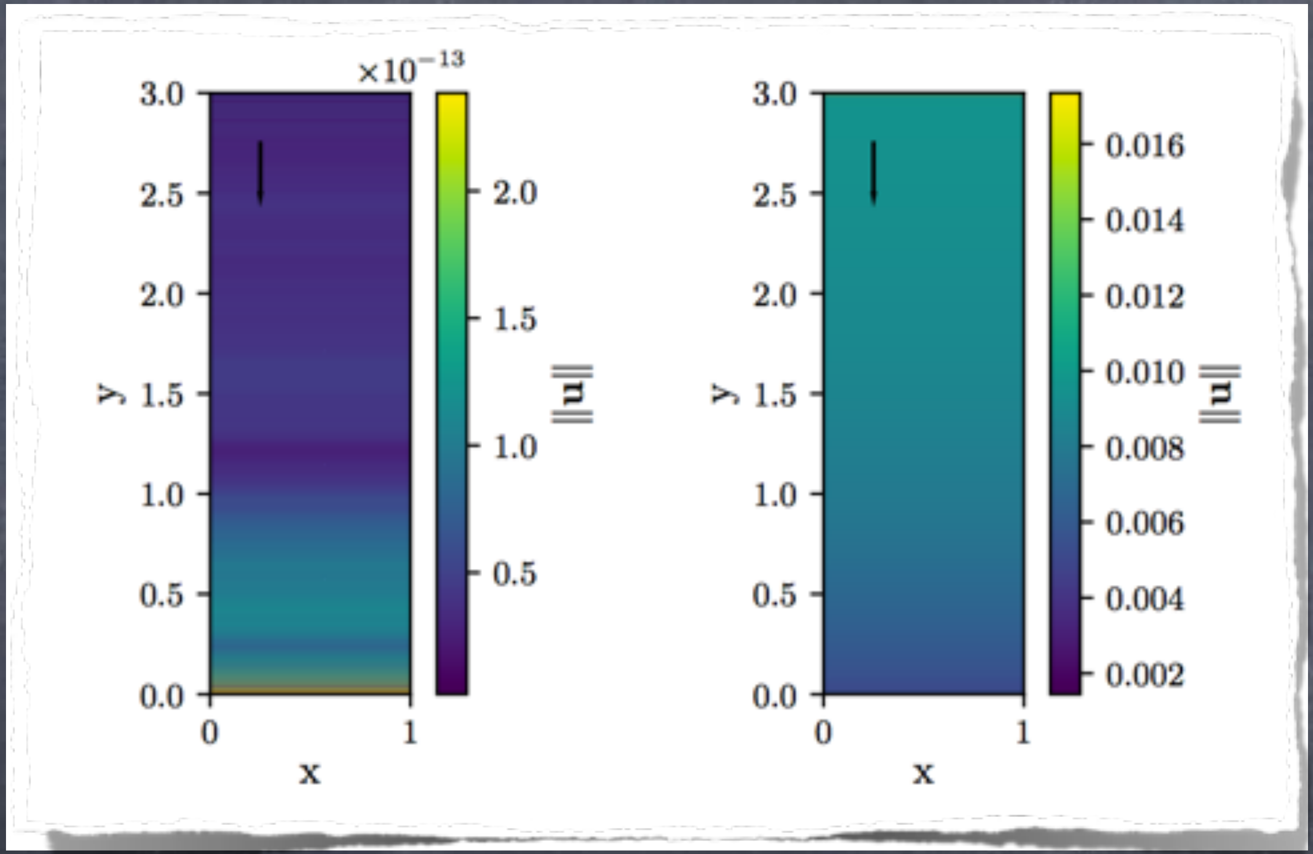
if there is initially no velocity

$$\frac{\Pi_i + \Pi_{i+1}}{2} - \frac{\Pi_i + \Pi_{i-1}}{2} = -\frac{1}{2}\left(\frac{\rho_i + \rho_{i+1}}{2} + \frac{\rho_i + \rho_{i-1}}{2}\right) g\Delta x$$

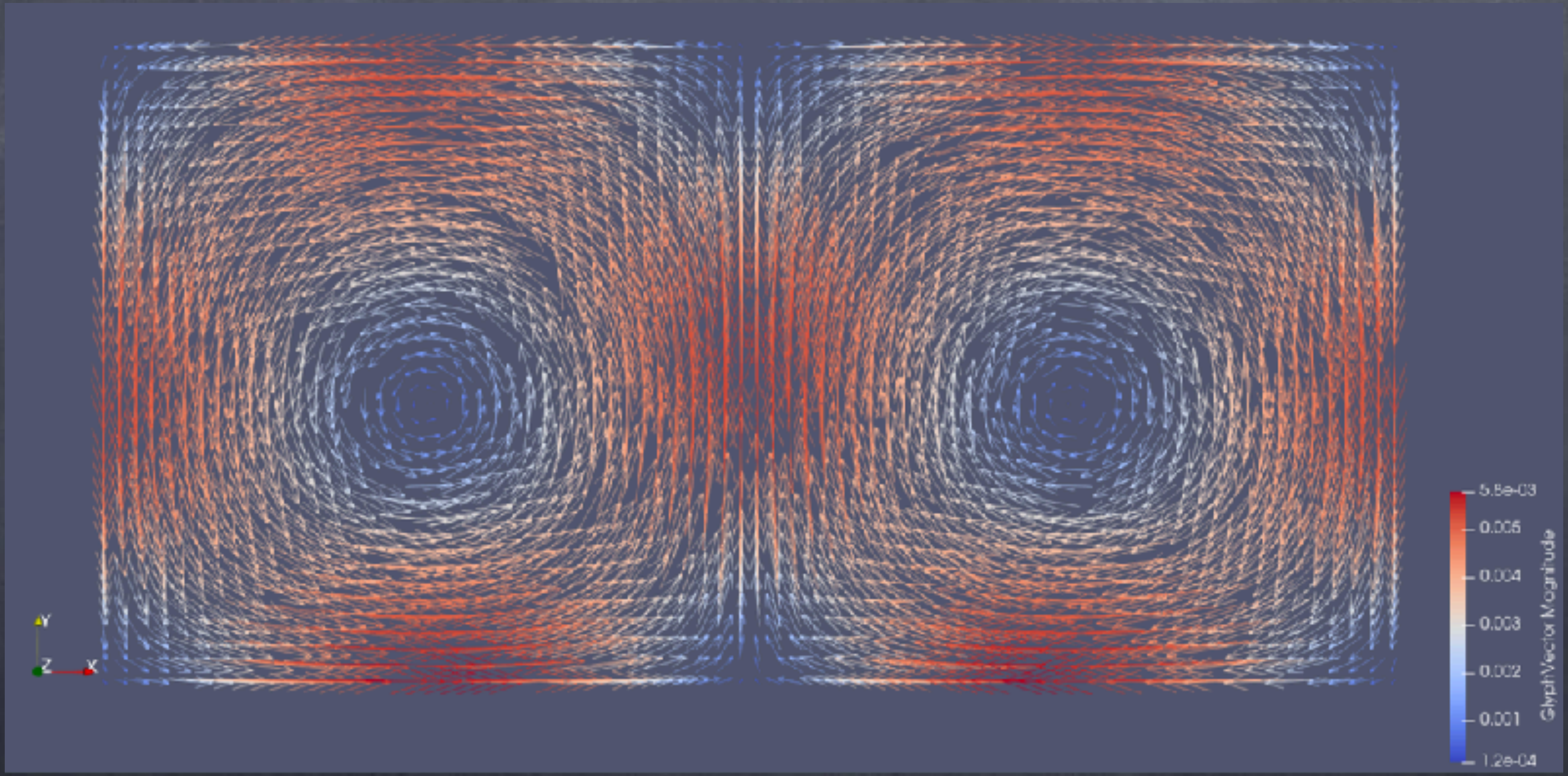
$$\Pi_{i+1} - \Pi_i = \frac{\rho_i + \rho_{i+1}}{2} g\Delta x \qquad \Pi_i - \Pi_{i-1} = \frac{\rho_{i-1} + \rho_i}{2} g\Delta x$$

$$u_x^{n+1} = 0, \quad \frac{\partial P}{\partial x} = -\rho g$$

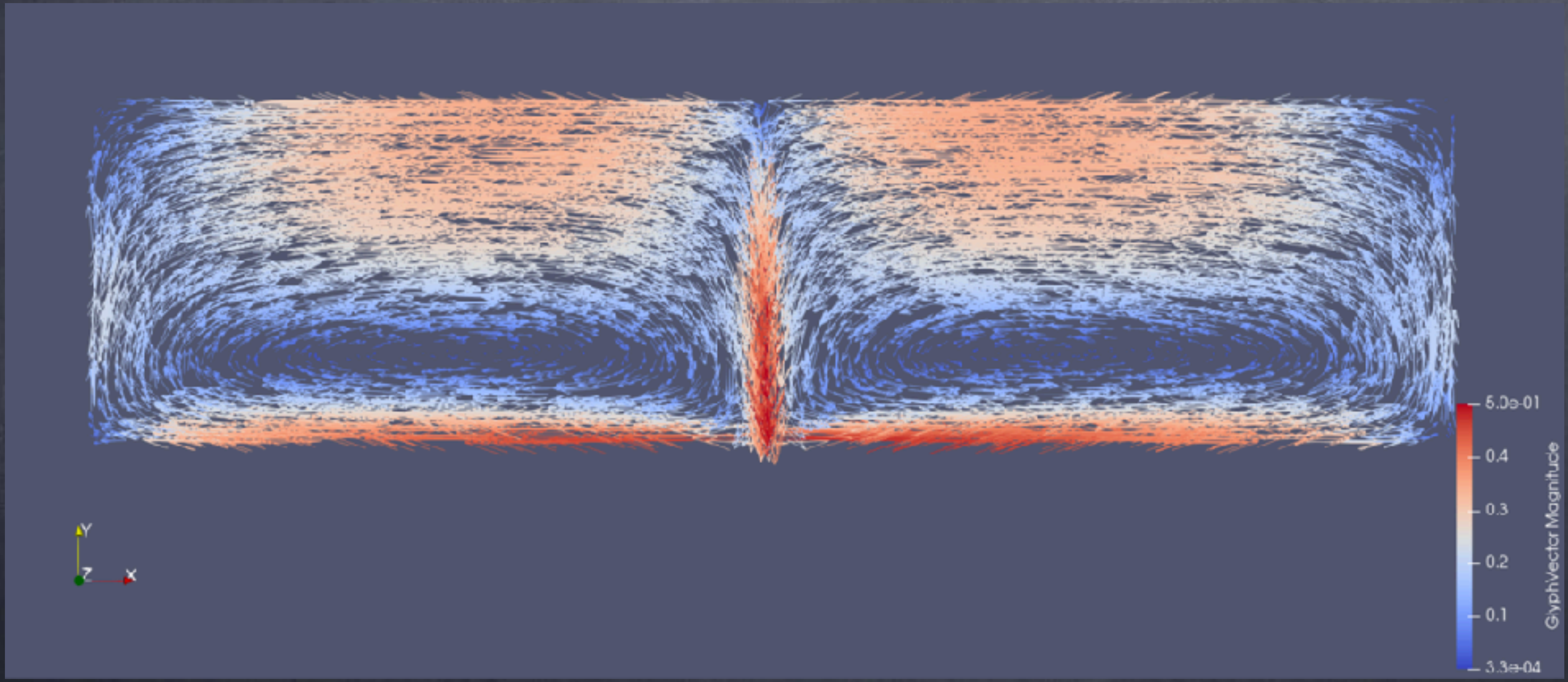
- Stratified compressible hydrodynamics
  - All-regime solver with gravity
    - hydrostatic balance at interface



- Stratified compressible hydrodynamics
  - All-regime solver with gravity
    - Convective simulation
    - Low Mach correction



- Stratified compressible hydrodynamics
  - All-regime solver with gravity
    - Compressible convective simulation



- Stratified compressible hydrodynamics
  - All-regime solver with gravity
    - Parallel HPC Implementation

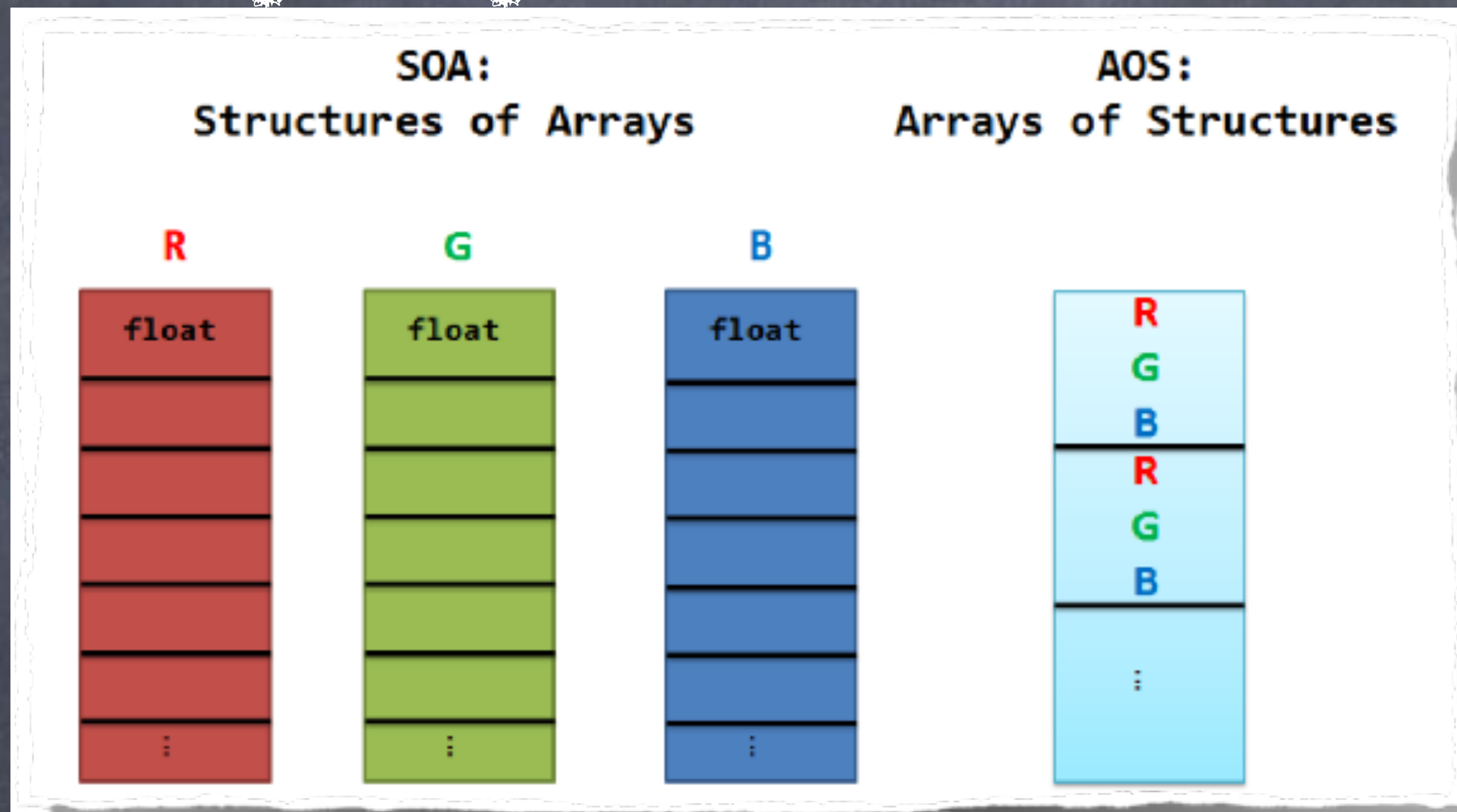
Problem of portability and performance probability...

- Stratified compressible hydrodynamics
  - All-regime solver with gravity
    - Parallel HPC Implementation

Kokkos library:

- C++ library for perf. portability
- extracted from Trilinos (Sandia)
- backend: openMP, Pthreads, CUDA
- abstraction of memory space and execution space

- Stratified compressible hydrodynamics
  - All-regime solver with gravity
    - Memory layout:



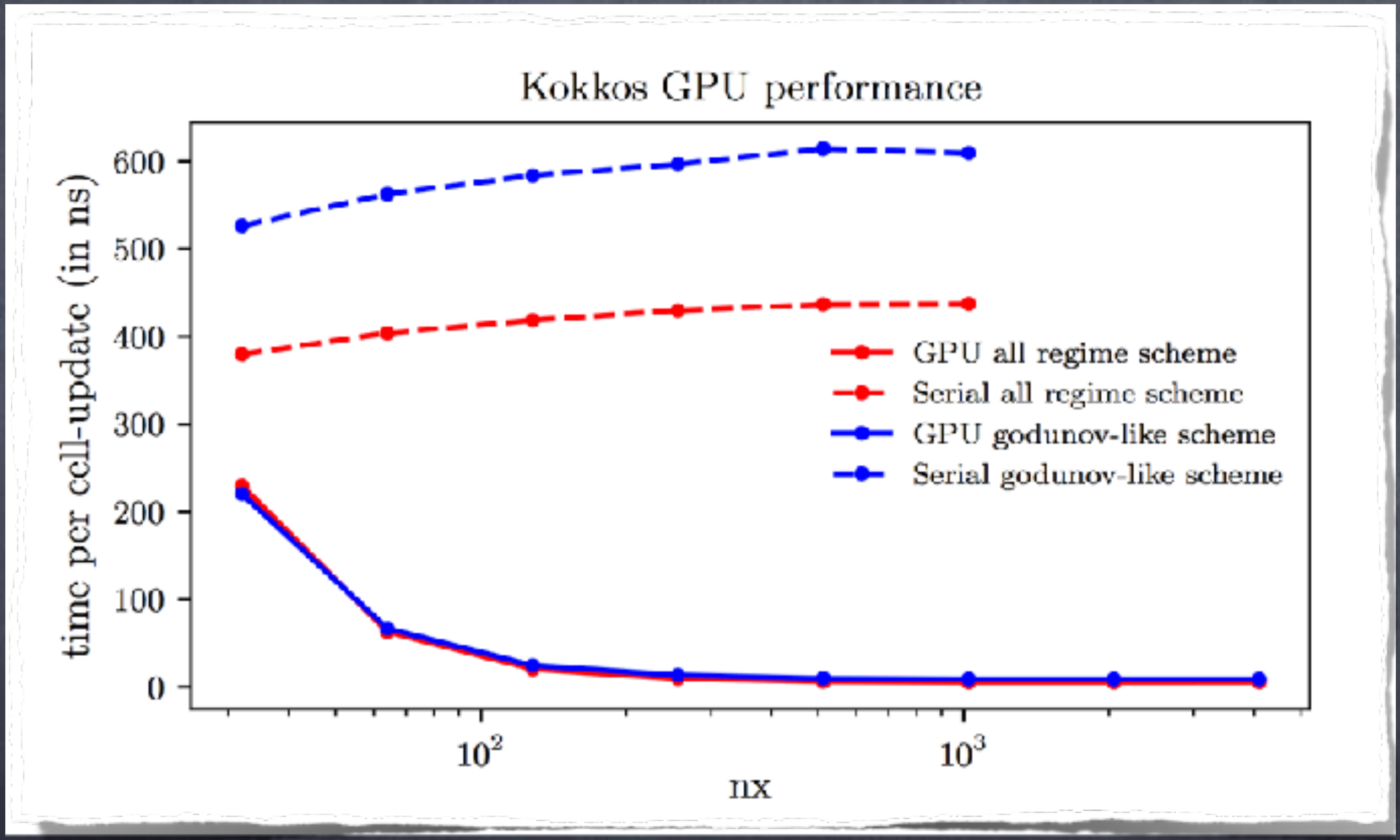

```
// Old matrix type:
// typedef View<double**,Device> my_matrix ;

// Change matrix type to an 8x8 tiled layout.
typedef View< double** ,
              LayoutTileLeft<8,8> ,
              Device >  my_matrix ;
```

- Stratified compressible hydrodynamics
  - All-regime solver with gravity
    - Kokkos kernel

```
struct InitView
{
  InitView(Kokkos::View<double*[3]> a)
    : m_a(a)
    {}

  KOKKOS_INLINE_FUNCTION
  void operator ()(const int i) const
  {
    a(i, 0) = 1.0*i;
    a(i, 1) = 1.0*i*i;
    a(i, 2) = 1.0*i*i*i;
  }

  Kokkos::View<double*[3]> m_a;
};
```

```
int main (int argc, char* argv[])
{

  Kokkos::initialize(argc, argv);

  Kokkos::View<double*[3]> view("View␣name", 15);

  Kokkos::parallel_for(15, InitView(view));

  Kokkos::finalize();

}
```

- Stratified compressible hydrodynamics
  - All-regime solver with gravity
    - Kokkos kernel



- Stratified compressible hydrodynamics
  - All-regime solver with gravity
    - 2D diabatic convection

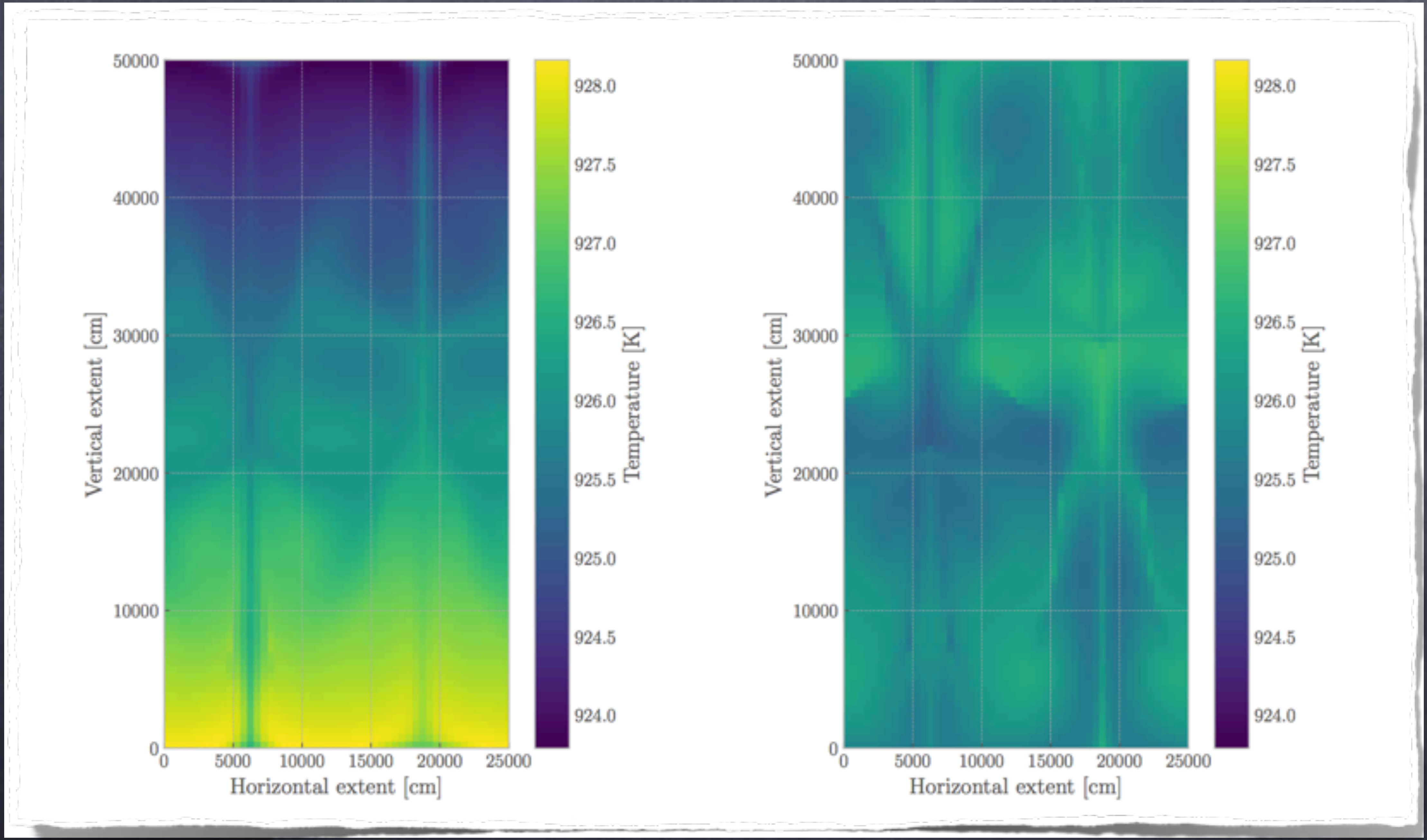
$$\frac{\partial \rho}{\partial t} + \overrightarrow{\nabla}\left(\rho \overrightarrow{u}\right) = 0$$

$$\frac{\partial \rho \overrightarrow{u}}{\partial t} + \overrightarrow{\nabla}\left(\rho \overrightarrow{u} \otimes \overrightarrow{u} + P\right) = \rho \overrightarrow{g}$$

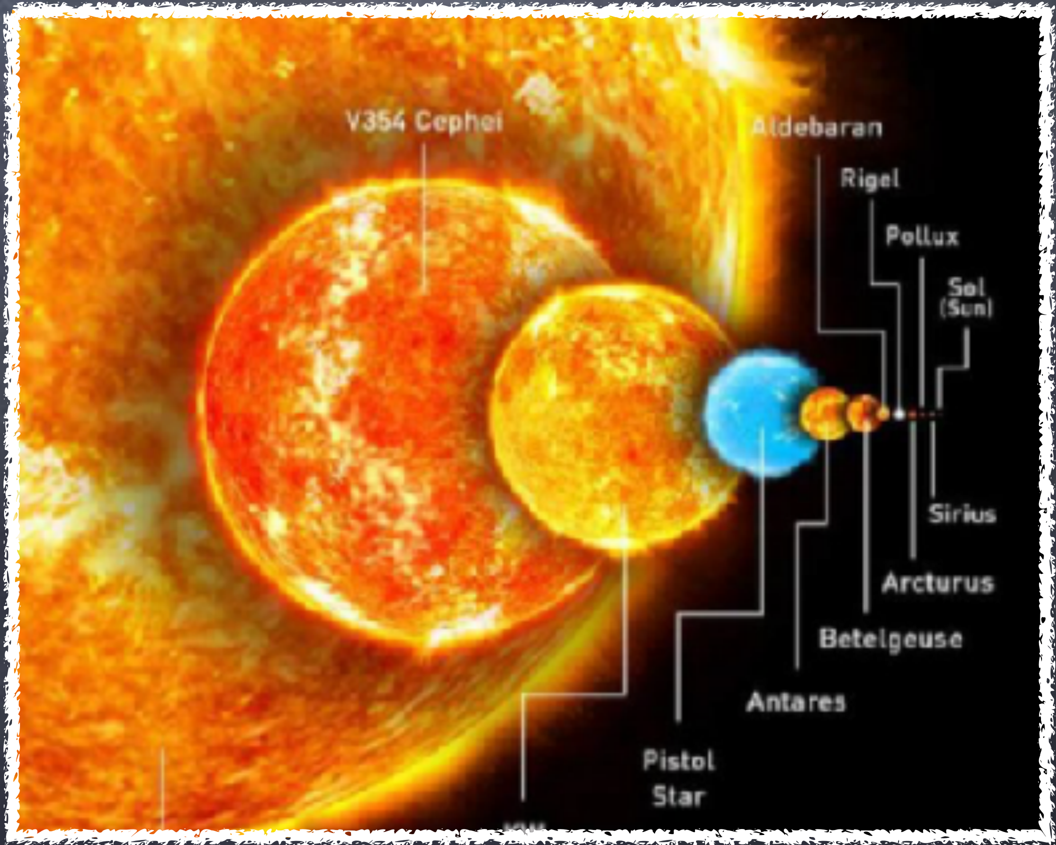
$$\frac{\partial \rho \mathcal{E}}{\partial t} + \overrightarrow{\nabla}\left(\overrightarrow{u}(\rho \mathcal{E} + P)\right) = \rho c_p H(X, T)$$

$$\frac{\partial \rho X}{\partial t} + \overrightarrow{\nabla}\left(\rho X \overrightarrow{u}\right) = \rho R(X, T)$$

- Stratified compressible hydrodynamics
  - All-regime solver with gravity
    - 2D diabatic convection



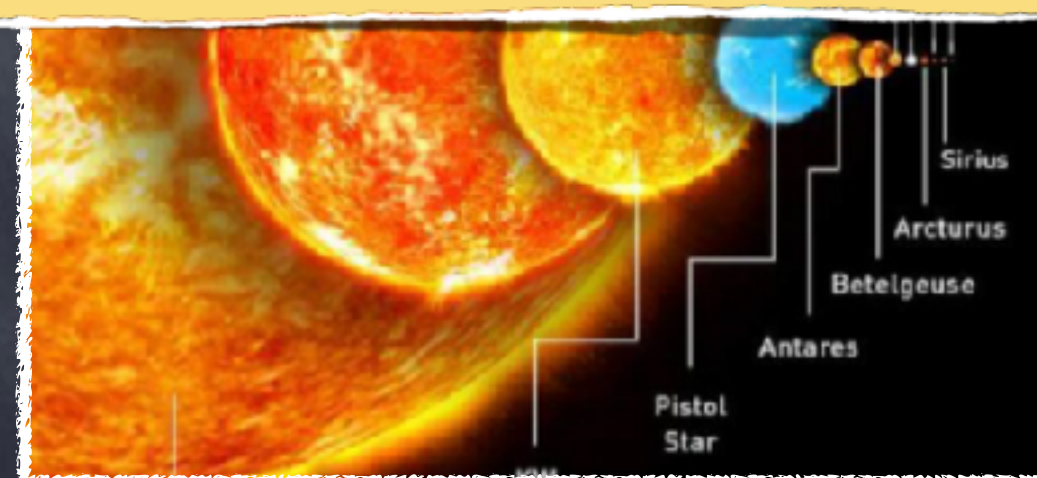
- What is in common between:



- What is in common between:

Convective systems but not adiabatic, they are all subject to:

- Energy exchange (latent heat, thermal diffusion, radiative transfer)
- and/or compositional source terms (chemical reactions, condensation/evaporation, compositional diffusion)



- What is adiabatic convection?
  - Thermal adiabatic case

$$\frac{\partial \ln \theta}{\partial t} + \overrightarrow{u}\,\overrightarrow{\nabla}\left(\ln \theta\right) = 0$$

$$\theta = T(P_{\mathrm{ref}}/P)^{(\gamma-1)/\gamma}$$

$$P = \rho k_b T/\mu$$

- What is adiabatic convection?
  - Thermal adiabatic case

$$\frac{\partial \ln \theta}{\partial t} + \overrightarrow{u}\,\overrightarrow{\nabla}\left(\ln \theta\right) = 0$$

$$\theta = T(P_{\mathrm{ref}}/P)^{(\gamma-1)/\gamma}$$

$$P = \rho k_b T/\mu$$

  - Unstable if: $\frac{\partial \ln \theta_0}{\partial z} < 0$

- Schwarzschild criterion (1906)

$$\nabla_T - \nabla_{\mathrm{ad}} > 0, \quad \nabla_T = \frac{\partial \ln T_0}{\partial \ln P_0}$$

$$\frac{\partial T_0}{\partial z} < \frac{g}{C_p}$$

- What is adiabatic convection?
  - Thermo-compositional adiabatic case

$$\frac{\partial \ln \theta}{\partial t} + \overrightarrow{u}\,\overrightarrow{\nabla}\left(\ln \theta\right) = 0$$

$$\frac{\partial X}{\partial t} + \overrightarrow{u}\,\overrightarrow{\nabla}(X) = 0$$

$$\theta = T(P_{\rm ref}/P)^{(\gamma-1)/\gamma}$$

$$P = \rho k_b T/\mu(X)$$

- What is adiabatic convection?
  - Thermo-compositional adiabatic case

$$\frac{\partial \ln \theta}{\partial t} + \vec{u}\,\vec{\nabla}\left(\ln \theta\right) = 0 \qquad \theta = T(P_{\rm ref}/P)^{(\gamma-1)/\gamma}$$

$$\frac{\partial X}{\partial t} + \vec{u}\,\vec{\nabla}(X) = 0 \qquad P = \rho k_b T/\mu(X)$$

  - Unstable if: $\nabla_T - \nabla_{\rm ad} - \nabla_\mu > 0$

$$\nabla_T = \frac{\partial \ln T_0}{\partial \ln P_0}, \quad \nabla_\mu = \frac{\partial \ln \mu_0}{\partial \ln P_0}$$

- Ledoux criterion (1947)

- What is adiabatic convection?
  - Thermo-compositional diabatic case

$$\frac{\partial \ln \theta}{\partial t} + \overrightarrow{u}\,\overrightarrow{\nabla}\left(\ln \theta\right) = \frac{H(X,T)}{T}$$

$$\theta = T(P_{\mathrm{ref}}/P)^{(\gamma-1)/\gamma}$$

$$\frac{\partial X}{\partial t} + \overrightarrow{u}\,\overrightarrow{\nabla}(X) = R(X,T)$$

$$P = \rho k_b T/\mu(X)$$

- What is adiabatic convection?
  - Thermo-compositional diabatic case

$$\frac{\partial \ln \theta}{\partial t} + \vec{u}\,\vec{\nabla}\left(\ln \theta\right) = \frac{H(X,T)}{T} \qquad \theta = T(P_{\rm ref}/P)^{(\gamma-1)/\gamma}$$

$$\frac{\partial X}{\partial t} + \vec{u}\,\vec{\nabla}\,(X) = R(X,T) \qquad P = \rho k_b T/\mu(X)$$

- Unstable if: $\nabla_T - \nabla_{\rm ad} - \nabla_\mu > 0$

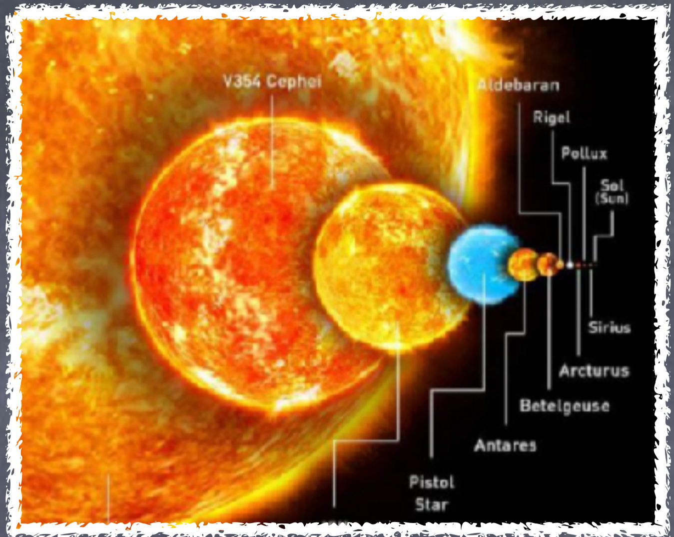
or

$$(\nabla_T - \nabla_{\rm ad})\omega'_X - \nabla_\mu \omega'_T < 0$$

with $\omega'_X = R_X + R_T(T_0 \partial \ln \mu_0/\partial X)$

and $\omega'_T = H_T + H_X(T_0 \partial \ln \mu_0/\partial X)^{-1}$

- Thermohaline or fingering convection



$$(\nabla_T - \nabla_{\rm ad})\omega'_X - \nabla_\mu \omega'_T < 0$$

with $\quad R = \kappa_\mu \Delta X \quad \omega'_X = -k^2\kappa_\mu \quad (R_T = 0)$

and $\quad H = \kappa_T \Delta T \quad \omega'_T = -k^2\kappa_T \quad (H_X = 0)$

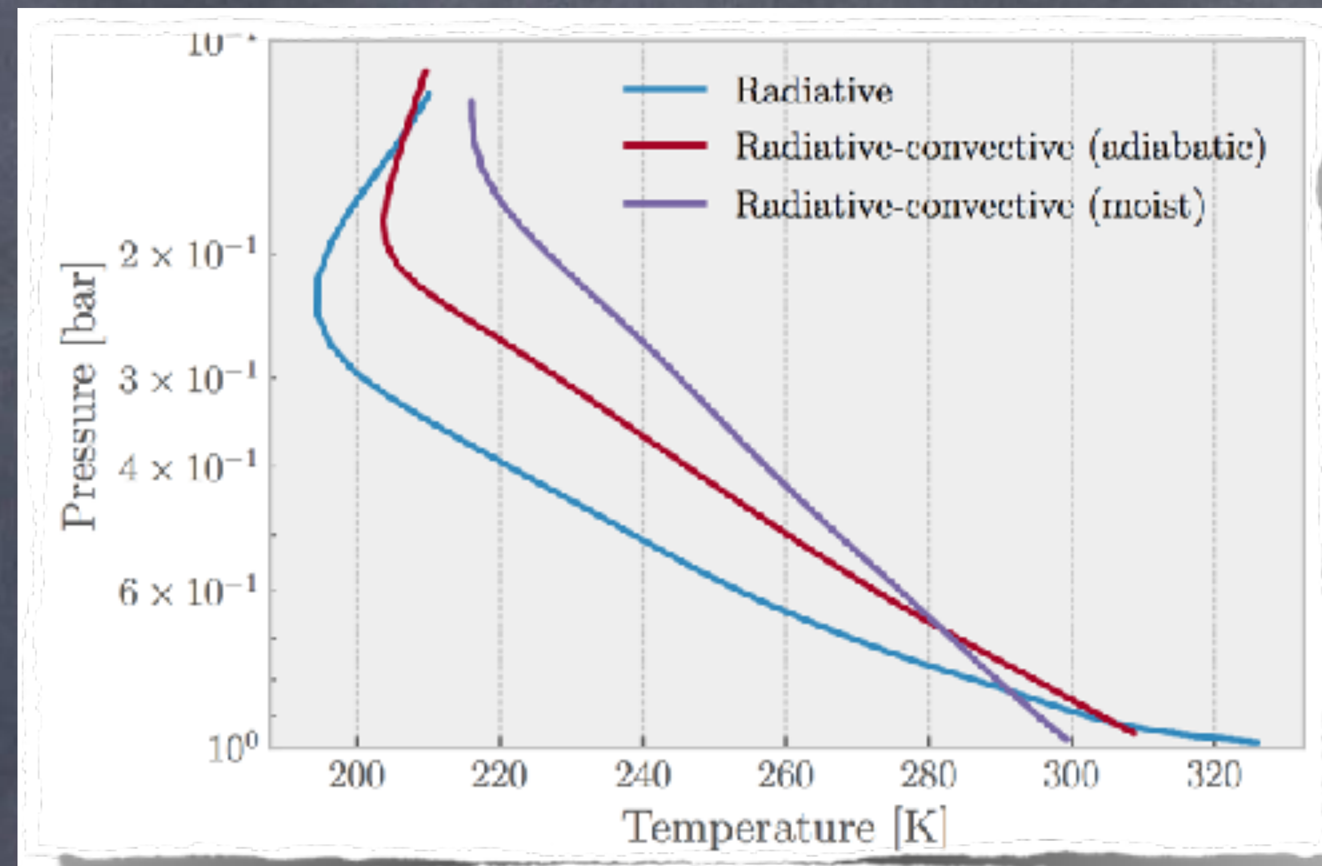
Stern 1960

Ulrich 1972

$$(\nabla_T - \nabla_{\rm ad})\kappa_\mu - \nabla_\mu \kappa_T > 0$$

and simulations from
Traxler et al. 2011, Brown et al. 2013
Garaud et al. 2015, Sengupta &
Garaud 2018

- Steam/liquid or moist convection



von Bezold 1893

$$\nabla_T - \nabla_{\rm ad} > 0$$

Dry adiabat

$$\nabla_T - \nabla_{\rm ad} \frac{1 + \frac{X_{\rm eq} L}{R_d T_0}}{1 + \frac{X_{\rm eq} L^2}{c_p R_v T_0^2}} > 0$$

Moist « pseudo-adiabat »

- Steam/liquid or moist convection

$$(\nabla_T - \nabla_{\rm ad})\omega'_X - \nabla_\mu \omega'_T < 0$$

$$\text{with} \quad R = R_{\rm cond}(X,T) \quad \text{and} \quad X = X_{\rm eq}(P,T)$$

$$\text{and} \quad H = -R_{\rm cond} L / c_p$$

von Bezold 1893

$$\nabla_T - \nabla_{\rm ad} \frac{1 - \rho_0 \frac{\partial X_{\rm eq}}{\partial P} L}{1 + \frac{\partial X_{\rm eq}}{\partial T} \frac{L}{c_p}} > 0$$

Moist « pseudo-adiabat »

- Steam/liquid or moist convection

$$(\nabla_T - \nabla_{\rm ad})\omega'_X - \nabla_\mu \omega'_T < 0$$

with $\quad R = R_{\rm cond}(X, T) \quad$ and $\quad X = X_{\rm eq}(P, T)$

and $\quad H = -R_{\rm cond} L / c_p$

von Bezold 1893

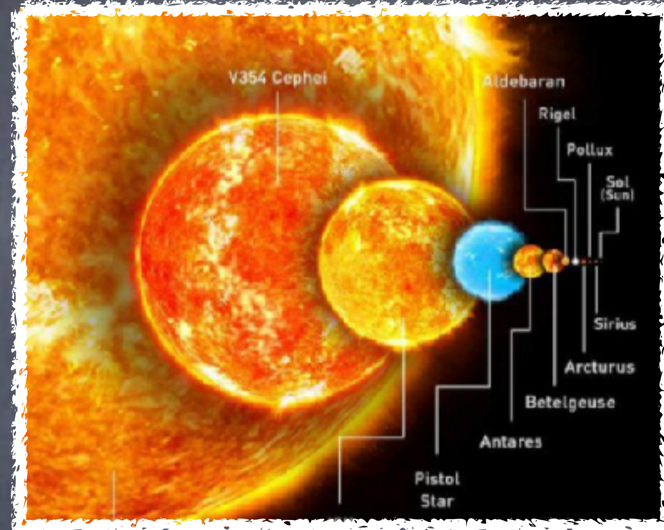
$$\nabla_T - \nabla_{\rm ad} \frac{1 - \rho_0 \dfrac{\partial X_{\rm eq}}{\partial P} L}{1 + \dfrac{\partial X_{\rm eq}}{\partial T} \dfrac{L}{c_p}} > 0$$

Moist « pseudo-adiabat »

- Thermo-compostional diabatic convection

Moist

Steam/liquid



Fingering

Thermohaline

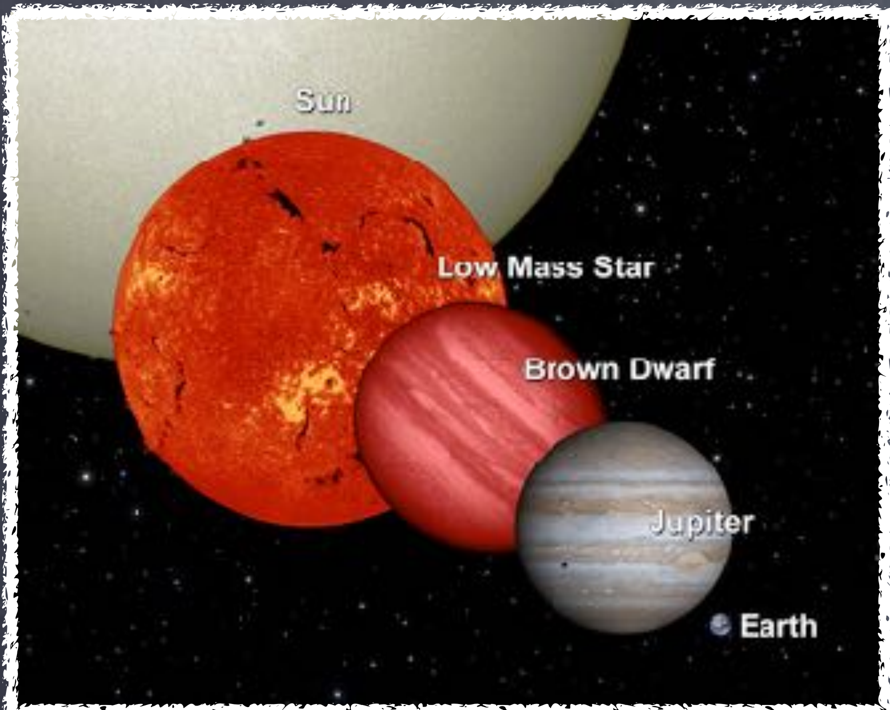
$$(\nabla_T - \nabla_{\mathrm{ad}})\omega'_X - \nabla_\mu \omega'_T < 0$$

with $\omega'_X = R_X + R_T(T_0 \partial \ln \mu_0 / \partial X)$

and $\omega'_T = H_T + H_X(T_0 \partial \ln \mu_0 / \partial X)^{-1}$

and probably many more...

- CO/CH4 radiative convection

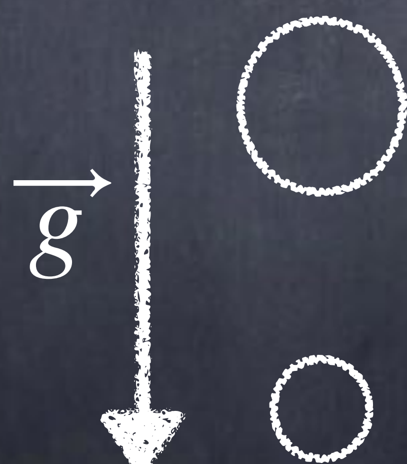


Brown dwarfs and giant exoplanets

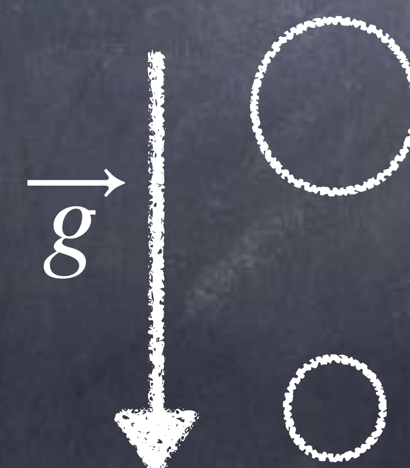
$$(\nabla_T - \nabla_{\mathrm{ad}})\omega'_X - \nabla_\mu \omega'_T < 0$$

with $\quad R = -(X - X_{\mathrm{eq}})/\tau_{\mathrm{chem}}$

and $\quad H = 4\pi\kappa/c_p\left(J - \sigma T^4\right)$



Moist convection



CO/CH4 radiative convection

- Generalisation of mixing length theory

$$\frac{\partial \ln \theta}{\partial t} + \overrightarrow{u}\overrightarrow{\nabla}\left(\ln \theta\right) = \omega'_T \frac{\delta T}{T_0}$$

$$\frac{\partial X}{\partial t} + \overrightarrow{u}\overrightarrow{\nabla}(X) = \omega'_X \delta X$$

$$\delta X \partial \ln \mu_0 / \partial X = \delta T / T_0$$

- Generalisation of mixing length theory

$$\frac{\partial \ln \theta}{\partial t} + \overrightarrow{u}\,\overrightarrow{\nabla}\left(\ln \theta\right) = \omega'_T \frac{\delta T}{T_0}$$

$$\frac{\partial \ln \theta'}{\partial t} + \overrightarrow{u}\,\overrightarrow{\nabla}\left(\ln \theta'\right) = 0$$

$$\frac{\partial X}{\partial t} + \overrightarrow{u}\,\overrightarrow{\nabla}(X) = \omega'_X \delta X$$

$$\delta X \partial \ln \mu_0 / \partial X = \delta T / T_0$$

$$\text{with} \quad \ln \theta' = \ln \theta - X \frac{\partial \ln \mu_0}{\partial X} \frac{\omega'_T}{\omega'_X}$$

- Generalisation of mixing length theory

$$\frac{\partial \ln \theta}{\partial t} + \overrightarrow{u}\,\overrightarrow{\nabla}\left(\ln \theta\right) = \omega'_T \frac{\delta T}{T_0}$$

$$\frac{\partial X}{\partial t} + \overrightarrow{u}\,\overrightarrow{\nabla}(X) = \omega'_X \delta X$$

$$\delta X \partial \ln \mu_0 / \partial X = \delta T / T_0$$

$$\frac{\partial \ln \theta'}{\partial t} + \overrightarrow{u}\,\overrightarrow{\nabla}\left(\ln \theta'\right) = 0$$

with $\ln \theta' = \ln \theta - X \dfrac{\partial \ln \mu_0}{\partial X} \dfrac{\omega'_T}{\omega'_X}$

$\ln \theta' = \ln \theta - XL/c_p T_0$ for moist convection

- Generalisation of mixing length theory

Can define an adiabatic convective flux:

$$F_{\rm ad} = \rho c_p w_{\rm ad} T_0 (\nabla_T - \nabla_{\rm ad})$$

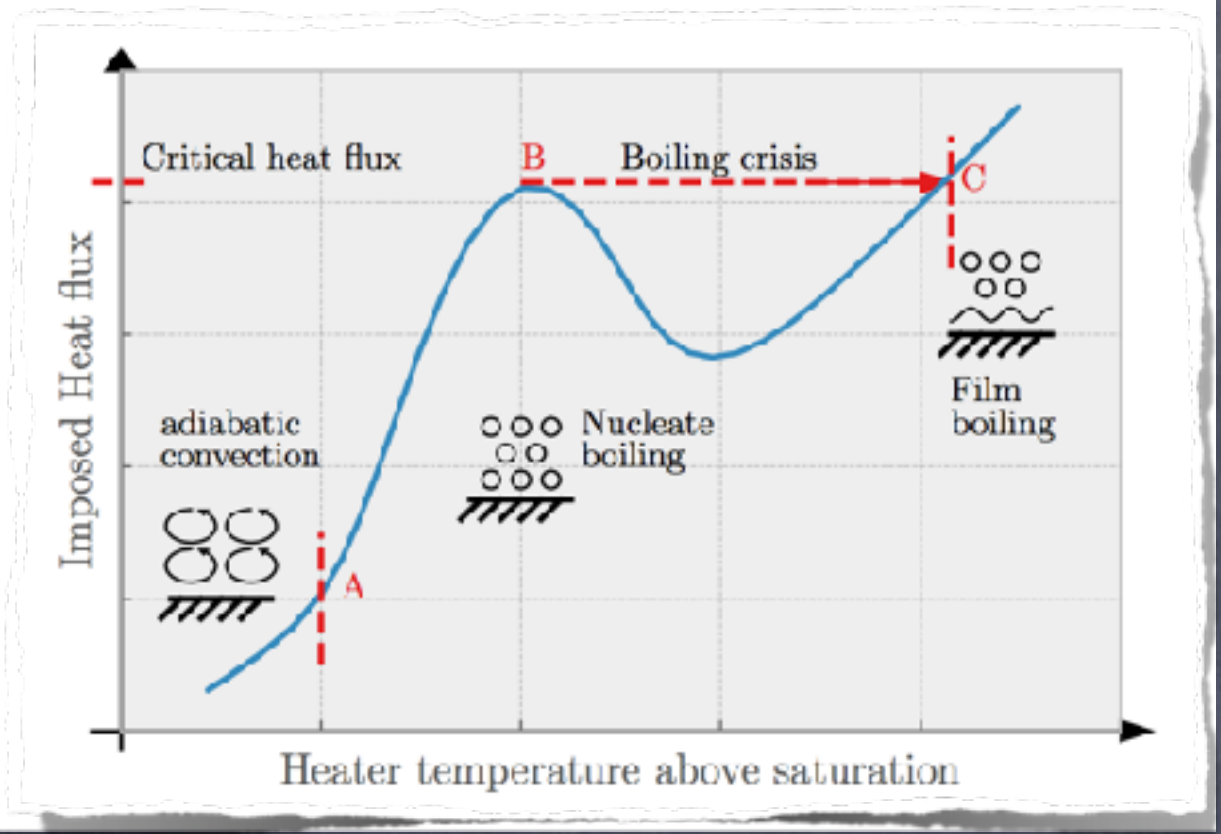
or

Can define a diabatic convective flux:

$$F_{\rm d} = \rho c_p w_{\rm d} T_0 (\nabla_T - \nabla_{\rm ad} - \nabla_\mu \omega'_T/\omega'_X)$$

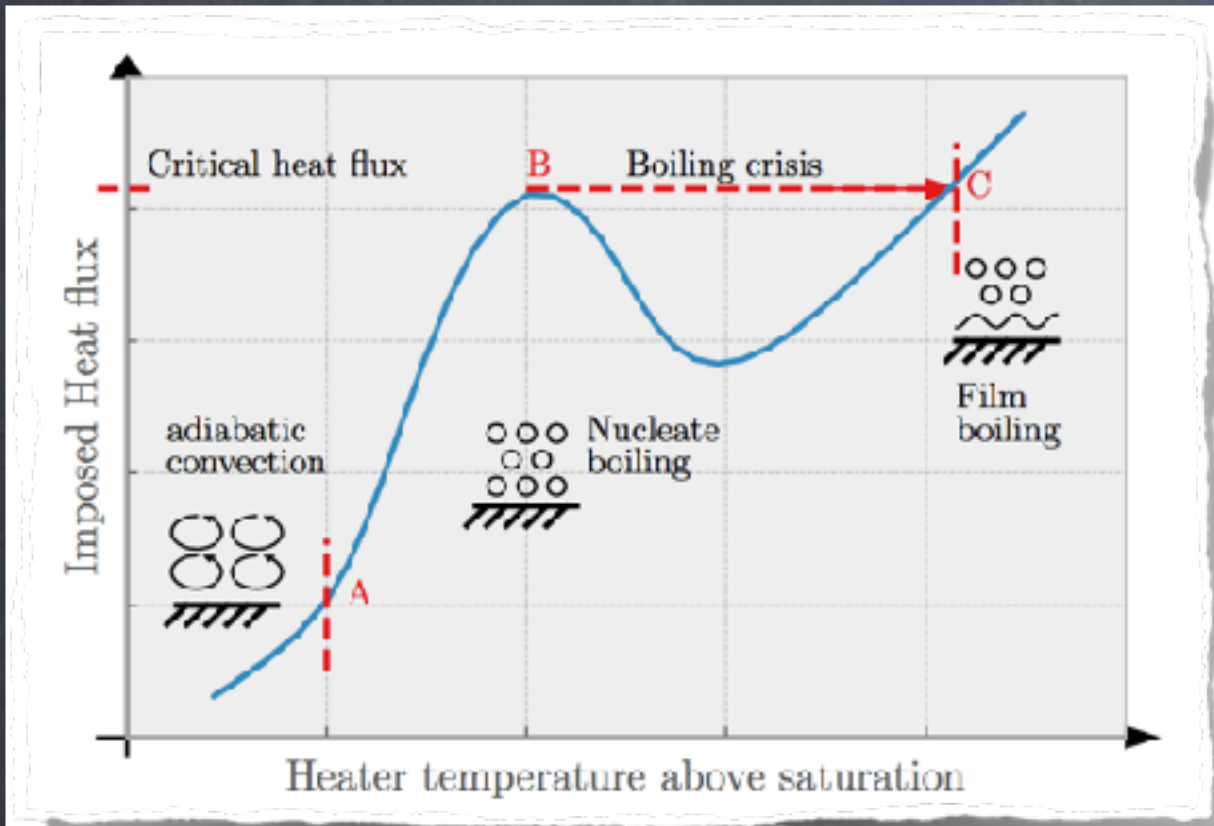
similar to mass/flux convection
parametrizations used for moist convection
(review: Arakawa & Jung 2011)

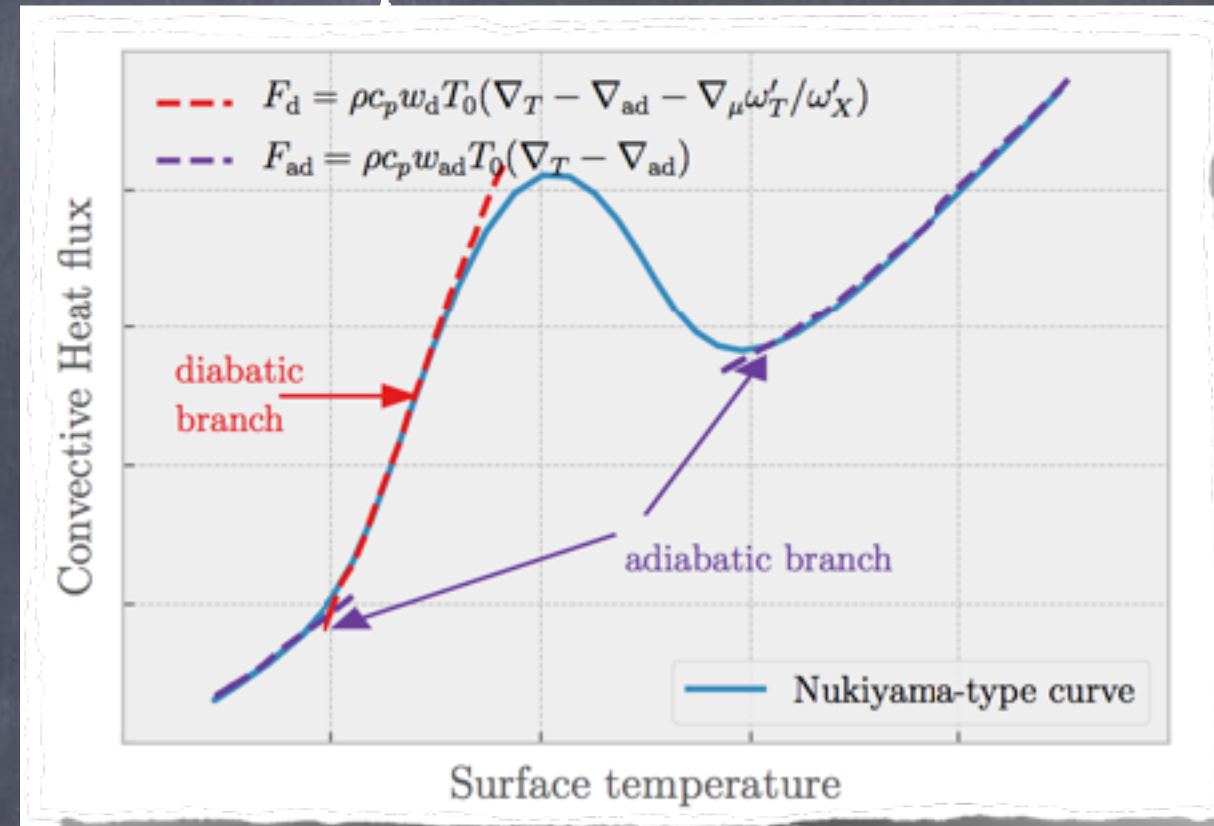
- Bifurcation between adiabatic and diabatic convection
  - Boiling crisis in steam/liquid convection



Nukiyama 1934

- Bifurcation between adiabatic and diabatic convection
  - Boiling crisis in steam/liquid convection

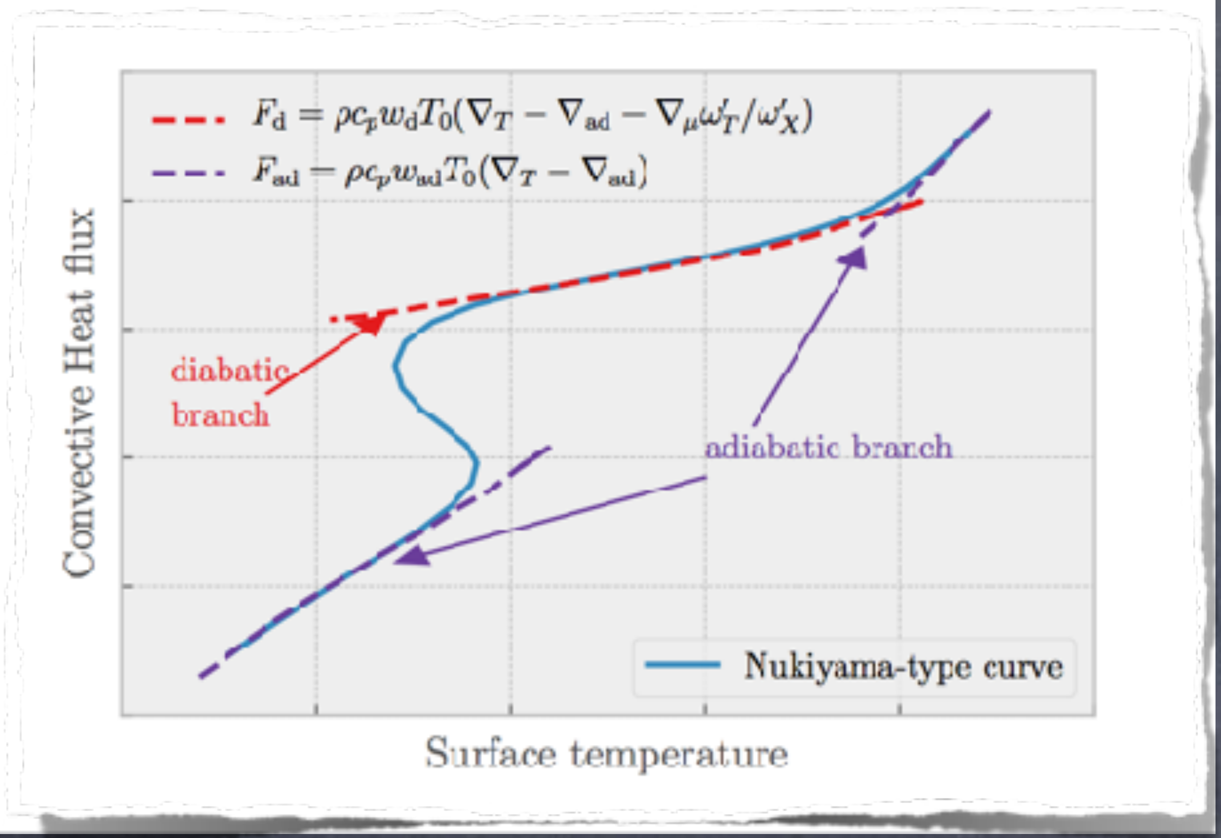


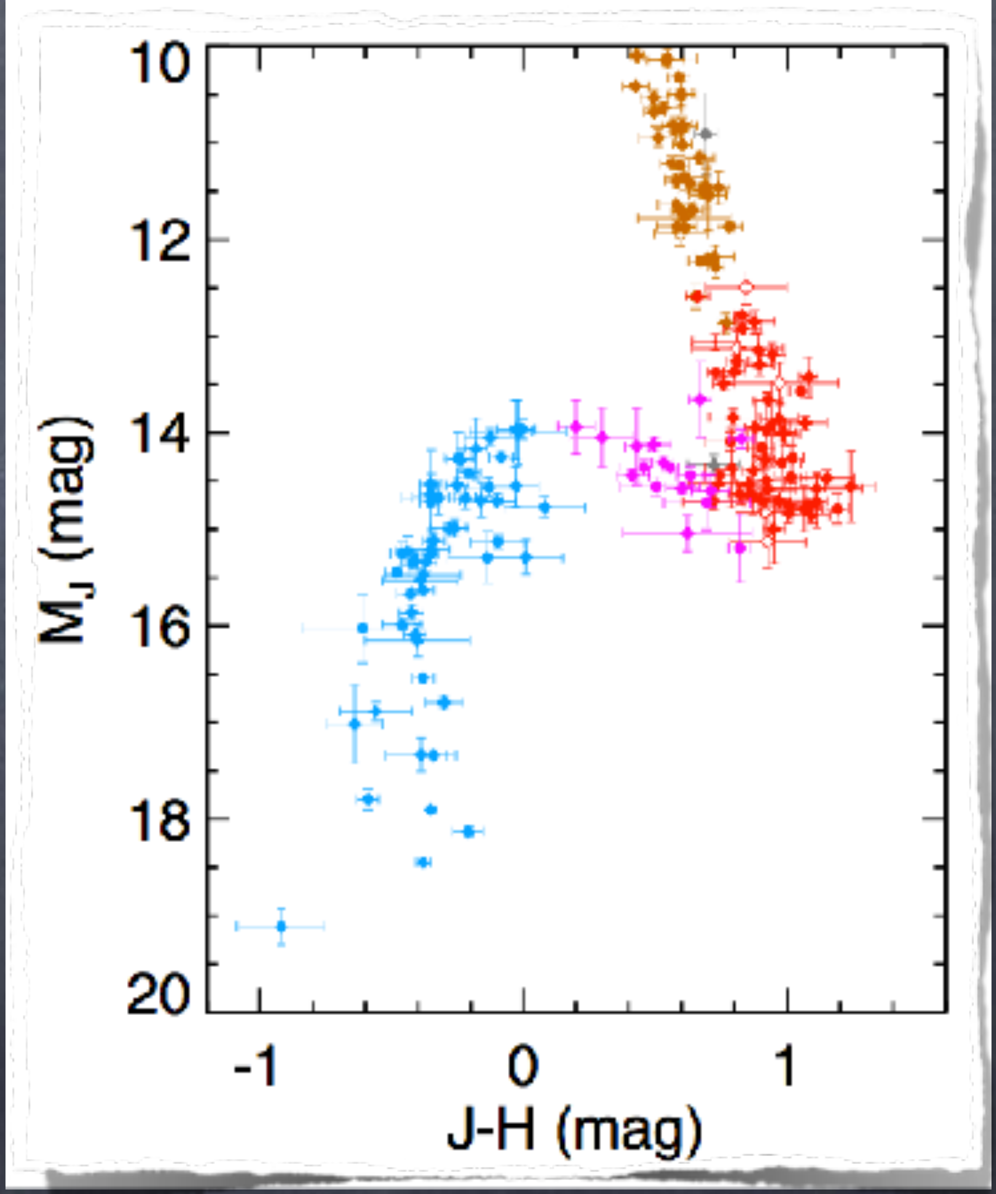


Nukiyama 1934

- Could provide a natural explanation of the boiling crisis?

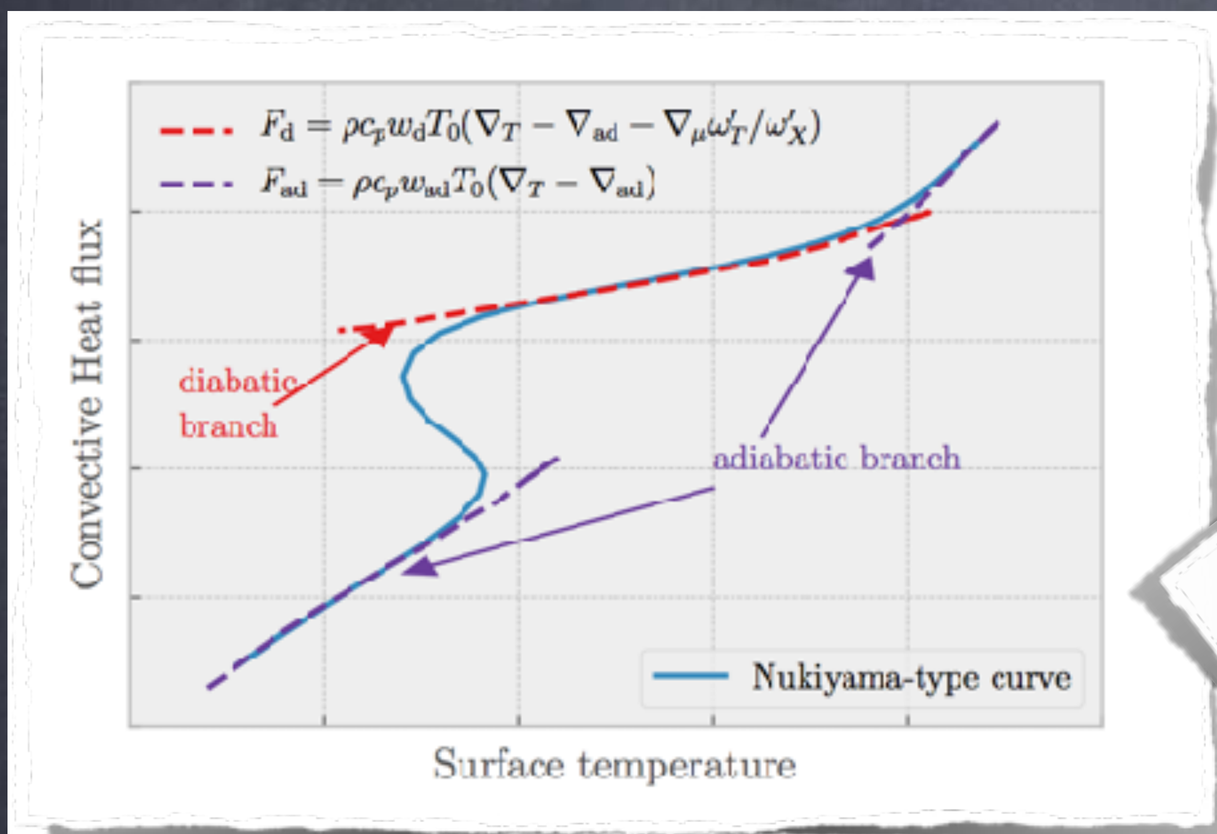
- Bifurcation between adiabatic and diabatic convection
  - L/T transition in brown-dwarf spectra?

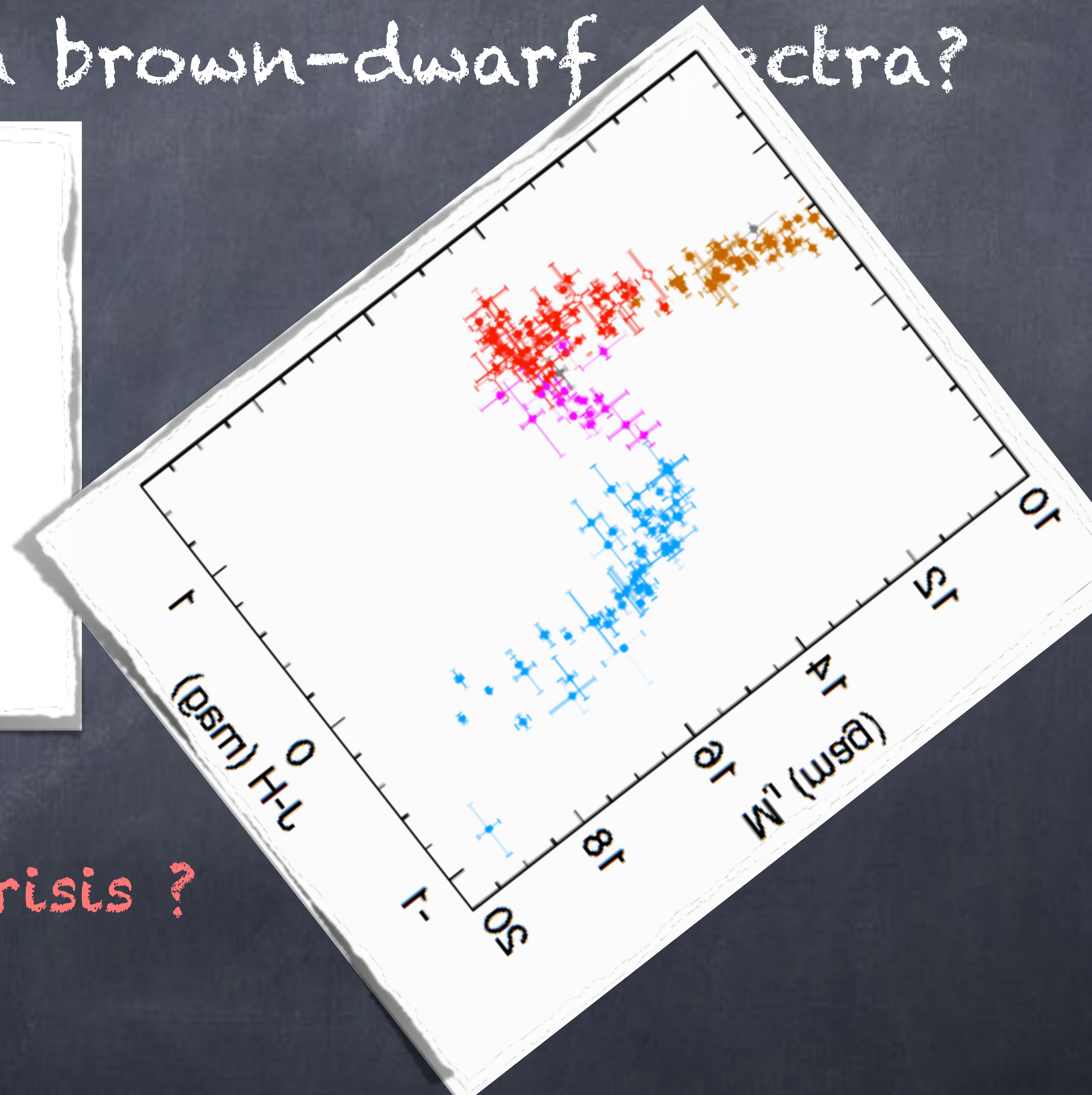




Dupuy & Liu 2012

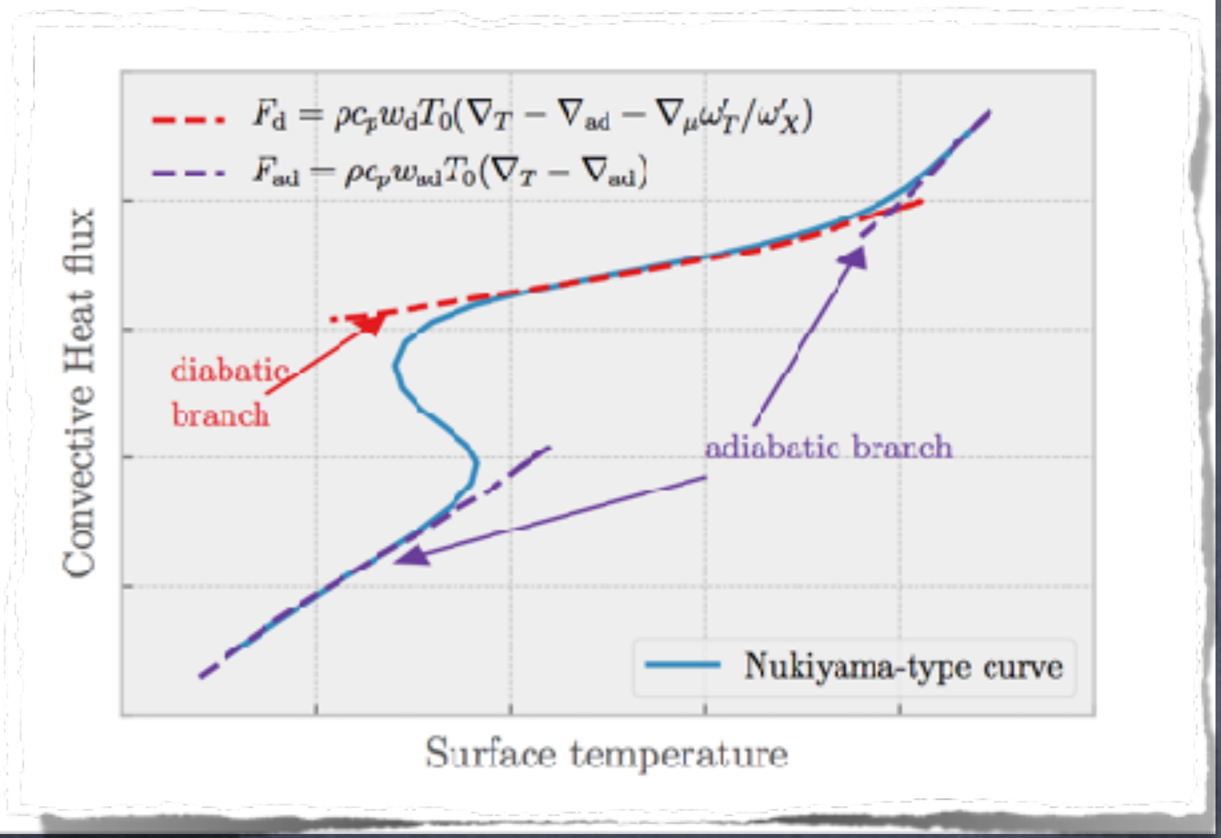
- Bifurcation between adiabatic and diabatic convection
  - L/T transition in brown-dwarf spectra?

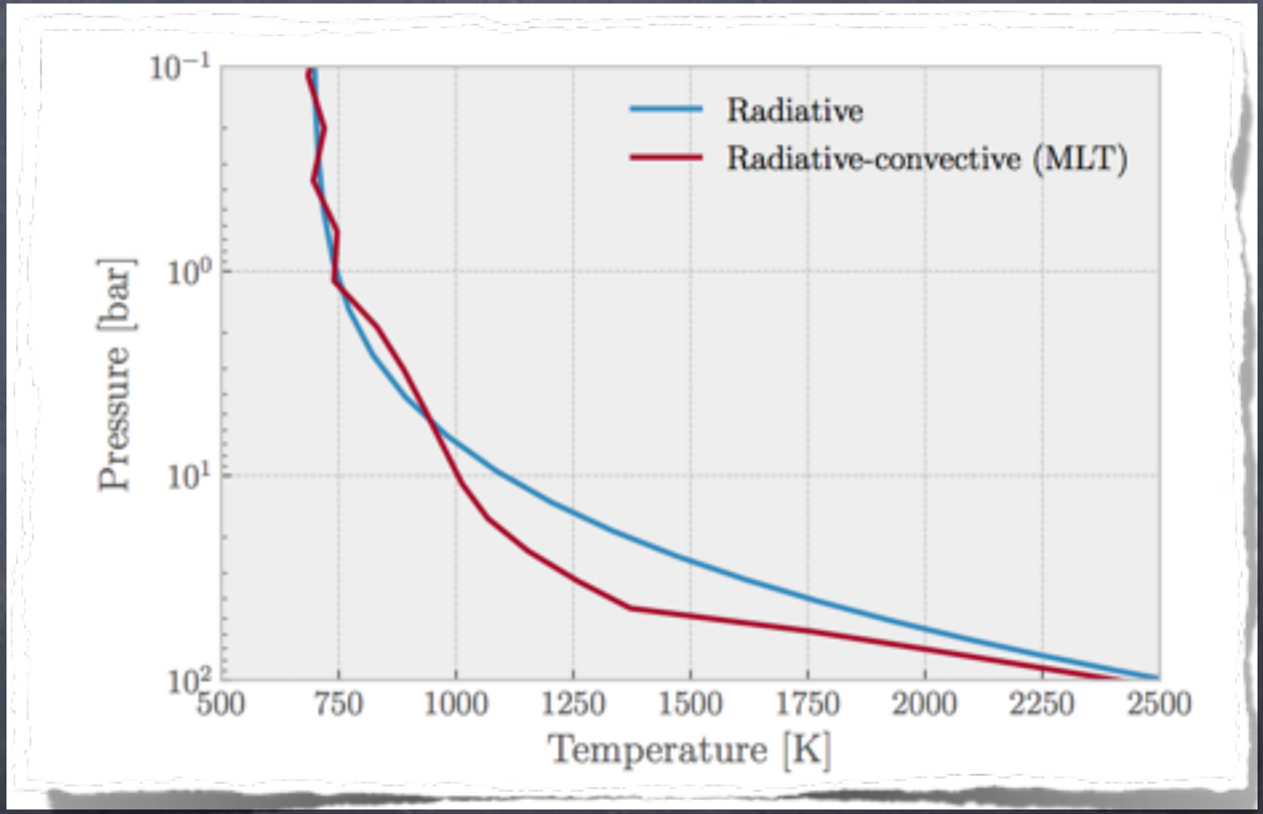




A giant cooling crisis ?

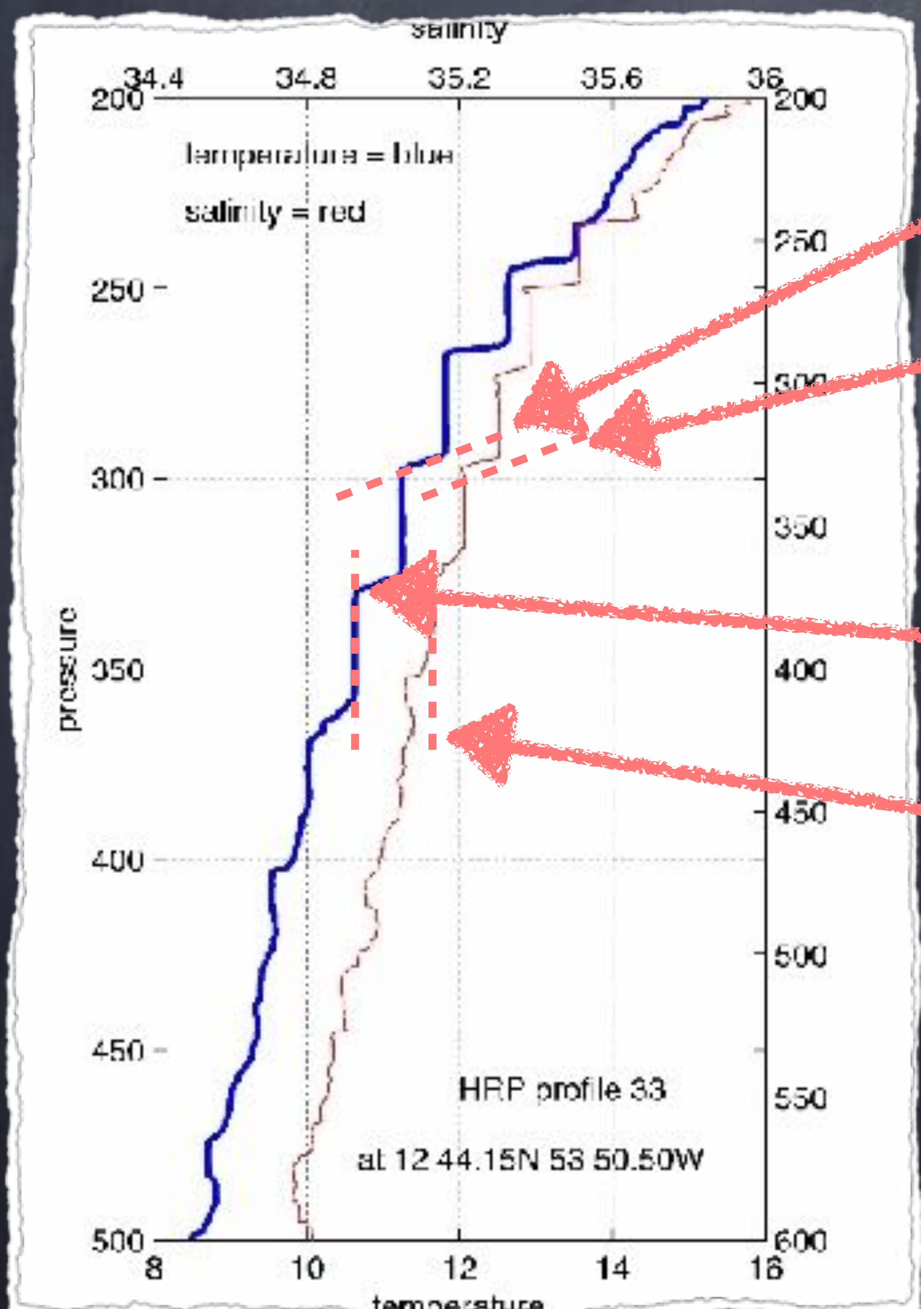
- Bifurcation between adiabatic and diabatic convection
  - L/T transition in brown-dwarf spectra?





- Bifurcation between adiabatic and diabatic convection

- Spatial bifurcation: thermohaline staircase



$$F_{\rm d} = \rho c_p w_{\rm d} T_0 (\nabla_T - \nabla_{\rm ad} - \nabla_\mu \kappa_T/\kappa_\mu)$$

$$X_{\rm d} = \rho w_{\rm d} \left(\frac{\partial \log \mu_0}{\partial X} \right)^{-1} (\nabla_\mu - (\nabla_T - \nabla_{\rm ad})\kappa_\mu/\kappa_T)$$

$$F_{\rm ad} = \rho c_p w_{\rm ad} T_0 (\nabla_T - \nabla_{\rm ad})$$

$$X_{\rm ad} = \rho w_{\rm ad} \left(\frac{\partial \log \mu_0}{\partial X} \right)^{-1} (\nabla_\mu)$$

- Conclusions:
  - Stratified all-regime compressible hydrodynamic solvers can be developed with efficient HPC support (MPI+Kokkos) to study these convective instabilities
  - Thermohaline, fingering, moist, steam/liquid, CO/CH4 radiative convection all derive from this diabatic branch of convection
  - Extension at high order, implicit acoustic waves and radiative transfer (Trilinos)