Cosmic Rays transport in the interstellar medium (ISM) : Role of the self-generated turbulence

Implementation of a sub-grid Cosmic Rays (CRs) diffusion coefficient and resonant Alfven waves drift velocity terms in the magnetohydrodynamics (MHD) RAMSES code

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Context

Cosmic Ray (CR) transport is closely linked to the turbulent dynamics of the interstellar medium (ISM). In this frame, an implicit scheme for solving the anisotropic diffusion of heat and CRs in the RAMSES code has been developed by Dubois & Commerçon (2016). The CR diffusion coefficient is initially constant throughout the simulation volume and evolves according to the turbulent properties of the gas (Lazarian 2016) throughout the simulation time.
We implement a routine allowing to correct the CRs diffusion coefficient value by considering the sub-resolution turbulent motions and observe the consequences at small/intermediary scales (> 40 pc). We also need to consider a streaming velocity term which takes in account the real resonant Alfvén waves velocity and their propagation direction along the magnetic field (Pfrommer et al. 2016).

Anisotropic diffusion of CRs in the RAMSES code

In order to take in account the CRs energy dynamics, Dubois & Commerçon (2016) implemented an equation of transport for CRs energy in the MHD RAMSES code.

The MHD mono-magnetized fluid equations are given by :

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \quad (1)$$

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot \left(\rho \mathbf{u}\mathbf{u} + p_{\rm tot} - \frac{\mathbf{B}\mathbf{B}}{4\pi} \right) = 0 \quad (2)$$

$$\frac{\partial e}{\partial t} + \nabla \cdot \left((e + p_{\rm tot})\mathbf{u} - \frac{\mathbf{B}(\mathbf{B}\cdot\mathbf{u})}{4\pi} \right) = \quad (3)$$

$$-\nabla \cdot \mathbf{F}_{\rm cond} - \nabla \cdot \mathbf{F}_{\rm CR} \quad (4)$$

$$\frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{u} \times \mathbf{B}) = 0 \quad (5)$$

where the source term $\nabla \cdot \mathbf{F}_{\rm CR}$ correspond to the energy tranfert rates between the fluid energy and CRs energy. The latter are considered as fluid leading to one more transport equation for the CRs energy density ($e_{\rm CR}$) :

$$\frac{\partial e_{\rm CR}}{\partial t} + \nabla \cdot (e_{\rm CR}) = -p_{\rm CR} \nabla \cdot \mathbf{u} - \nabla \cdot \mathbf{F}_{\rm CR}. \quad (6)$$

$\mathbf{F}_{\rm CR} = -D_{\rm CR}\mathbf{b}(\mathbf{b}\cdot\nabla)e_{\rm CR}$ where $D_{\rm CR}$ is constant over all the simulation box.

Turbulence and CRs in MCs

Constant diffusion coefficient

One can define a critical diffusion coefficient $D_{\rm crit}$ above which dynamical effect of CRs is withdrawn :

$$D_{\rm crit} = 2.5 \times 10^{23}\ {\rm cm}^2{\rm s}^{-1} \left(\frac{L_{\rm box}}{1\ {\rm pc}} \right)^{1.5}. \quad (7)$$

This coefficient is based on the Larson relations which relate the non-thermal velocity fluctuations $\Delta v_{\rm NT}$ to the size and mass of MCs. Following the scaling given by Hennebelle & Falgarone (2012), Commerçon et al. (in prep. 2018) derive

$$\Delta v_{\rm NT} \sim 1\ {\rm km\ s}^{-1} \left(\frac{L}{1\ {\rm pc}} \right)^{0.5} \quad (8)$$

$$n \sim 3000\ {\rm cm}^{-3} \left(\frac{L}{1\ {\rm pc}} \right)^{-0.7} \quad (9)$$

where L is the size of the regions in which the non-thermal velocity and the density are measured. The critical diffusion coefficient $D_{\rm crit}$ is derived by equating the characteristic turbulent time ($t_{\rm turb} = L_{\rm box}/\Delta v_{\rm NT}$) with the diffusion time ($t_{\rm diff} = L_{\rm box}^2/D$).
The figure 1. represents the volume-weighted Probability Density Function (PDF) of the gas density, temperature and CR energy density. Different simulations have been performed : one with $D_0 = 0$ (No-diffusion) and without CRs, the others with CRs without diffusion and for a constant background diffusion coefficient evolving from 10^{22} to 10^{28} cm^2 s^{-1}. The density and temperature PDFs are wider in models with $D_0 \geq 10^{26}$ cm^2 s^{-1}. For the CRs we observe the contrary, the CRs PDFs are wider for low values of diffusion coefficient suggesting that a too high value imply that large diffusion coefficients smooth CR gradients and prevent the development of kinetic CR driven instabilities.

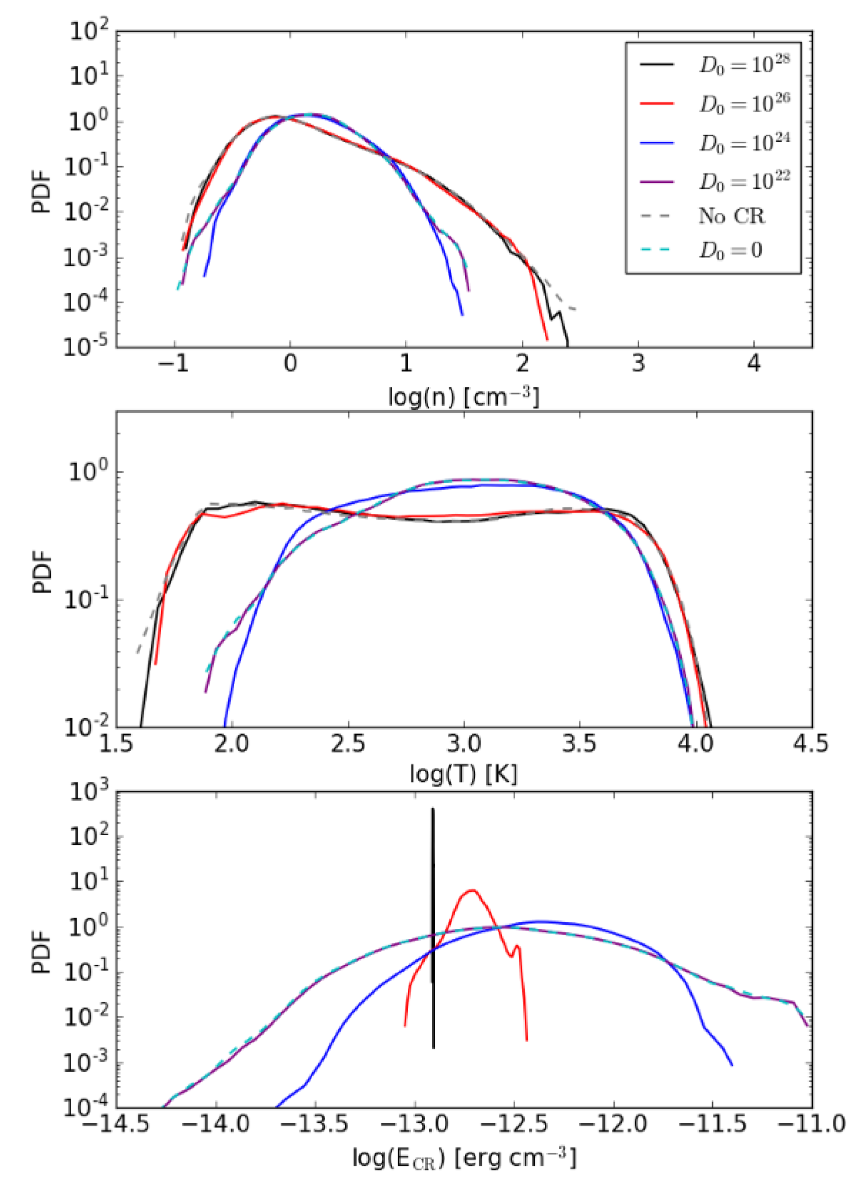

FIGURE 1 – PDF of the gas density (top), temperature (middle), and CR energy density (bottom), for $D_0 = 10^{22}$ (purple), 10^{24} (blue), 10^{26} (red), and 10^{28} cm^2 s^1 (black). Two models with $D_0 = 0$ (dashed cyan) and without CR (dashed grey) are also represented for comparison.

However, the critical value of the CRs diffusion coefficient can be reached by considering more sophisticated models of turbulence :

→ Large scale turbulence model based on the Alfvenic regime. The turbulence properties are widely dependant of the structural properties of MCs (Yan & Lazarian 2008).

→ CRs self-generated turbulence model. CRs streaming can generate magnetic perturbations which contribute to the global turbulence and decreases the total CRs diffusion coefficient.

Diffusion coefficient of CRs in MCs

Commerçon et al. (in prep. 2018) implemented a more realistic value of the CRs diffusion coefficient $D_{\rm CR}$ based on different large scale turbulence models. Cho et al. (2002) isolate different regimes of MHD turbulence depending on the Alfvénic Mach number ($M_A = V_L/V_A$ where V_L is the turbulent rms velocity of the fluid and V_A is the Alfvén velocity). Based on this model, Yan & Lazarian (2008) propose a derivation of the CR diffusion coefficients induced by these large-scale-injected turbulent motions. This coefficient will be noted D_0 hereafter :

→ **For super-Alfvénic turbulence** ($M_A > 1$) :
- CRs diffusion in super-Alfvénic turbulence is isotropic $D = D_\parallel = D_\perp$ but depends on their parallel mean free path $\lambda_\parallel$.

$\lambda_\parallel > L_{\rm inj}$	$\lambda_\parallel < L_{\rm inj}$
$D = l_A v/3$	$D = \lambda_\parallel v/3$

→ **For sub-Alfvénic turbulence** ($M_A < 1$) :
- $D_\parallel \gg D_\perp$ where $D_\parallel = \lambda_\parallel v/3$

$\lambda_\parallel > L_{\rm inj}$	$\lambda_\parallel < L_{\rm inj}$
$D_\perp = M_A^4 L_{\rm inj} v/3$	$D_\perp = D_\parallel M_A^4$

CRs self-generated diffusion coefficient

In order to consider the effect of the CRs self-generated turbulence in the turbulent box, we need to derive a diffusion coefficient $D_{\rm self}$ which will be added to the large scale diffusion coefficient D_0. The CRs global diffusion coefficient is then given by :

$$D_{\rm CR} = [D_0^{-1} + D_{\rm self}^{-1}]^{-1}. \quad (10)$$

The diffusion coefficient is calculated as follows :

→ We retrieve eqs. (4) and (5) from Nava et al. (2016) but assuming that $\partial I/\partial t + V_{\rm A}\,\partial I/\partial z = 0$ implying

$$-2\Gamma_{\rm in} I(k) + |V_{\rm st}| \left| \frac{\partial P_{\rm CR}(p)}{\partial s} \right| + Q(k) = 0 \quad (11)$$

Where $I(k)$ represent the magnetic perturbations energy density at a scale k^{-1}. We fix p and k, $\partial P_{\rm CR}/\partial s$ represent the pressure gradient of CRs for a given energy. We consider resonating Alfvén waves and we neglect the effect of the large scale turbulence $\Rightarrow Q(k) = 0$.

→ For each cell, we calculate $\Gamma_{\rm in}$ by solving the dispersion relation of the Alfvén waves.

→ For each cell, we calculate the CRs pressure gradient along the magnetic field line.

→ From eq. (11) we derive the wave energy density for each cell.

→ The parallel propagating diffusion coefficient is then given by : $D_{\rm self,\parallel} = 4\pi r_g c/(3I(k))$ while the perpendicular propagating diffusion coefficient is given by $D_{\rm self,\perp} = D_{\rm self,\parallel} I(k)^2$ in the quasi-linear limit.

Resonant Alven waves drift velocity

The eq. (6) can be rewritten in a conservative form

$$\frac{\partial e_{\rm CR}}{\partial t} + \nabla(e_{\rm CR}\mathbf{u} + \mathbf{F}_{\rm CR}) = -P_{\rm CR}\nabla \cdot \mathbf{u}. \quad (12)$$

But the spatial transport of CR energy density is advected with the frame propagating at the velocity $\mathbf{u} + \mathbf{v}_{\rm st}$ where $\mathbf{v}_{\rm st} = -V_A\ {\rm sgn}(\mathbf{B}\cdot\nabla P_{\rm CR})$ is the streaming velocity and corresponds to the Alfvén waves propagation velocity. The above eq. should be rewritten as

$$\frac{\partial e_{\rm CR}}{\partial t} + \nabla(e_{\rm CR}(\mathbf{u} + \mathbf{v}_{\rm st}) + \mathbf{F}_{\rm CR}) = -P_{\rm CR}\nabla \cdot (\mathbf{u} + \mathbf{v}_{\rm st}) \quad (13)$$

where $P_{\rm CR} = (\gamma_{\rm CR} - 1)e_{\rm CR}$.
The implementation of the streaming term can be done with two ways :

→ **Implicit way :**
In this case, the eq. above can be rewritten as

$$\frac{\partial e_{\rm CR}}{\partial t} + \nabla(e_{\rm CR}\mathbf{u}) + \gamma_{\rm CR}\nabla(e_{\rm CR}\mathbf{v}_{\rm st}) = -\nabla \cdot \mathbf{F}_{\rm CR} - P_{\rm CR}\nabla \cdot \mathbf{u} + \mathbf{v}_{\rm st} \cdot \nabla P_{\rm CR}$$

where the first red term can be treated as a streaming diffusion term ($\nabla(\mathbf{v}_{\rm st} e_{\rm CR}) = \nabla(D_{\rm st}\mathbf{b}\cdot\nabla e_{\rm CR})$ where $D_{\rm st} = V_A e_{\rm CR}/|\nabla e_{\rm CR}|$. The second red term correspond to a heating term and can be treated as a source term.

→ **Explicit way :**
In this case, the eq. above can be rewritten as

$$\frac{\partial e_{\rm CR}}{\partial t} + \nabla(e_{\rm CR}\mathbf{u}) + \gamma_{\rm CR} e_{\rm CR}\nabla\mathbf{v}_{\rm st} = -\nabla \cdot \mathbf{F}_{\rm CR} - P_{\rm CR}\nabla \cdot \mathbf{u} - \mathbf{v}_{\rm st} \cdot \nabla e_{\rm CR}$$

The numerical implementation of the streaming terms requires to regularize the streaming velocity to avoid discontinuities. In the explicit method, we use the upwind method (see Sharma et al. 2011) with a minmod slope limiter.

Numerical simulations of the CRs self-generated turbulence (in prep.) :

→ Source-less simulation of CRs diffusion with a fixed background diffusion coefficient : Understand the value effect of the background diffusion coefficient on the CRs diffusion without sources

→ Static presence of a CRs overpressure source (ex : SNR)

→ Static presence of more than one CRs overpressure source (ex : more than one SNR)

→ Time dynamical evolution of CRs overpressures which appear and desappear (ex : ISM SNs dynamics simulation)

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